

CSC236 tutorial exercises, Week #8

best before Friday afternoon

For each of the algorithms in questions 1-3, prove termination. If proving termination requires a loop invariant, you may state it without proof. (Though you should be confident that your invariant actually holds, and comfortable with how it *could* be proved, if necessary.)

1.

```
1 def ssum(A):
2     """Pre: A is a list of non-negative ints
3     Post: return the sum of A
4     WARNING: A may be irrevversibly altered!
5     """
6     i = 0
7     s = 0
8     while i < len(A):
9         if A[i] == 0:
10            i += 1
11        else:
12            s += 1
13            A[i] -= 1
14    return s
```

2.

```
1 def binsearch(A, x):
2     """Pre: A is a sorted list of numbers. x is a number.
3     Post: return i such that A[i] = x, or -1 if x is
4           not an element of A.
5     """
6     lo = 0
7     hi = len(A)-1
8     while lo <= hi:
9         m = lo + (hi - lo) // 2
10        mid = A[m]
11        if mid == x:
12            return m
13        elif mid < x:
14            lo = m + 1
15        else:
16            hi = m - 1
17    return -1
```

3.

```
1 def perambulate(A):
2     """Pre: A is a non-empty list of non-negative ints
3     """
4     seen = []
5     curr = A[0]
6     i = 0
7     while curr not in seen:
8         i = (i + curr) % len(A)
9         seen.append(curr)
10        curr = A[i]
11    return curr
```

4. Identify the logical flaw in the following “proof” of termination of the function mean.

```
1 def mean(a, b):
2     """Pre: a and b are ints, a < b
3     Post: return the arithmetic mean of a and b
4     """
5     while a != b:
6         a += 1
7         b -= 1
8     return a
```

Proof of termination Define loop measure $m_j = b_j - a_j$.

$m_j \in \mathbb{N}$, since $a < b$, by the precondition.

It remains to show that m decreases with each iteration. For an arbitrary iteration $j > 0$, $m_j = m_{j-1} - 2$ (since we increase a by 1 and decrease b by 1).

Thus $\langle m_0, m_1, \dots \rangle$ is a decreasing sequence of natural numbers, and therefore finite, so mean terminates.

□