CSC236 tutorial exercises, Week #4

best before Friday afternoon

Please attempt to solve these exercises *before* tutorial on Friday. During tutorial, your TA will answer questions about the problems, and review solutions to selected problems. These exercises are not graded, so you are encouraged to work with classmates on them, and discuss them on the course discussion board.

1. Consider the following code implementing binary search (you should be familiar with this algorithm from CSC148):

```python
def binsearch(A, x):
    """Return i such that A[i] = x.
    PRECONDITION: A is a non-empty sorted list, and x is an element of A.
    """
    if len(A) == 1:
        return 0
    mid = len(A) // 2
    if A[mid] > x:
        return binsearch(A[:mid], x)
    else:
        return binsearch(A[mid:], x) + mid
```

   (a) Devise a recurrence $T(n)$ that describes the worst-case number of steps taken by `binsearch` on input of size n (i.e. having `len(A) = n`). As usual, you may assume that n is a 'nice' size so that you can avoid floors and ceilings when dividing the input up into sublists (i.e., in this case, $n = 2^k$ for some $k \in \mathbb{N}$).

   (b) Use the technique of unwinding (AKA repeated substitution) to find a closed form for $T(n)$. You are welcome to use either of the techniques shown in lecture - either repeatedly expanding an algebraic expression, or drawing out a tree of recursive calls, and reasoning about the total depth, and number of steps taken at each level. Verify your closed form by testing it on a small value of n. (You don't need to prove it.)

2. Consider the following recurrence defined over $\mathbb{N}^+$ (the positive naturals):

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ n + 4T(n/4) & \text{if } n > 1 \end{cases}$$

   Use induction to prove the closed form $T(n) = n \log_4 n + n$ holds for all powers of 4.

   (I suggest using complete induction with the predicate $P(n) : n \text{ is a power of } 4 \implies T(n) = n \log_4 n + n$, but it can also be done using simple induction, if you do the induction on a different variable.)