CSC236 tutorial exercises, Week #10
best before Friday afternoon

1. Let BOOKENDS $= \{x \in \{a,b\}^* \mid |x| > 0 \wedge x[0] = x[-1]\}$, i.e. the language of non-empty strings where the first symbol matches the last symbol. Design (i.e. draw) a DFA that accepts BOOKENDS.

2. Consider the following formal definition of a DFA, $M = (Q, \Sigma, \delta, s, F)$, where

   - $Q = \{q_0, q_1, q_2\}$
   - $\Sigma = \{a, b\}$
   - $s = q_0$
   - $F = \{q_2\}$

   And the transition function δ is given by the following table (with states on the left, and symbols on top):

δ	a	b
q_0	q_1	q_0
q_1	q_2	q_1
q_2	q_2	q_2

   (a) Give a short English description of $\mathcal{L}(M)$, the language accepted by M.

   (b) Complete the following predicate definition by defining invariants for each of M's three states. Your invariants should imply the correctness of your description from part (a). (Recall that δ^* is the extended transition function):

   $$P(x) : \delta^*(s, x) = \begin{cases} q_0 & \text{if } \_\_\_\_\_\_ \\ q_1 & \text{if } \_\_\_\_\_\_ \\ q_2 & \text{if } \_\_\_\_\_\_ \end{cases}$$

   The conditions on x that you fill in should be exhaustive (so that the "if"s are equivalent to "iff").

   (c) Use induction to prove $\forall x \in \{a,b\}^*, P(x)$.

3. Suppose we have a DFA, $M = (Q, \Sigma = \{0, 1\}, \delta, s, F)$, such that for every state $q \in Q$, $\delta(q, 0) = \delta(q, 1)$. What can we conclude about $\mathcal{L}(M)$, the language accepted by M?