CSC236 tutorial exercises, Week #1
best before Friday afternoon

Please attempt to solve these exercises *before* tutorial on Friday. During tutorial, your TA will answer questions about the problems, and review solutions to selected problems. These exercises are not graded, so you are encouraged to work with classmates on them, and discuss them on the course discussion board.

1. Define $P(n)$ as:

$$\sum_{i=0}^{i=n} 2^i = 2^{n+1}$$

   (a) Prove that $P(115)$ implies $P(116)$.

   (b) Is $P(n)$ true for every natural number n? Explain why, or why not.

2. Use induction to prove that $\forall n \in \mathbb{N}$, $8^n - 1$ is a multiple of 7.

3. Use induction to prove that for every power of 7, there is a power of 3 with the same units digit.

4. Consider an alternative to our familiar inductive proof structure in which we prove the following:

$$P(0) \tag{1}$$
$$P(1) \tag{2}$$
$$\forall n, m \in \mathbb{N}, P(n) \wedge P(m) \implies P(n+m) \tag{3}$$

   Is this a valid proof that P holds for all natural numbers?

   (a) Use simple induction with the facts above to prove $\forall n \in N, P(n)$.

   (b) If we omit claim (3) above, obviously we can't conclude anything more profound than $P(0) \wedge P(1)$. But what numbers can we conclude that P holds for if we...

      i. Omit (1)?

      ii. Omit (2)?

      iii. Replace (1) and (2) with $P(2)$ and $P(3)$?