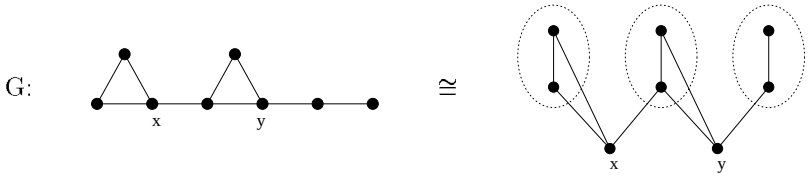
CSC 2410F - Introduction to Graph Theory

Assignment #2 Due: Oct. 31, 2002

NB. Read the policy on "grading scheme" in the course outline. All work must be your own; reference all sources.

1. Two vertices x, y are called *pseudo-similar* if $G - x \cong G - y$ but there is no automorphism mapping x onto y. For example,



   Can a tree have pseudo-similar vertices? Prove or disprove.

2. A k-dominating set is a subset of vertices of cardinality k such that every vertex not in the set is adjacent to at least one vertex in the set. Using the fact that the k-dominating set problem is NP-complete for general graphs, show that the problem is also NP-complete for bipartite graphs.

3. (a) Prove that if an interval graph has diameter 2 then there is a universal clique.

   (b) Does the same statement hold for chordal graphs? If so, give a proof; if not, give a counter-example.

4. Prove that a graph is of treewidth k iff it is a partial k-tree.

5. Consider an LBFS σ of graph G. Recall that a *slice* of an LBFS is a set of vertices that all have the lexicographically largest label (i.e. the algorithm has to choose an element from this set). Given two vertices x, y where $x <_\sigma y$, define $\Gamma^\sigma_{x,y}$ to be the smallest slice containing x and y.

   (a) Prove the following claim:

   Suppose $x <_\sigma y$. Let t be the first vertex of the connected component of $\Gamma^\sigma_{x,y}$ containing x. If t is distinct from x, then t cannot be adjacent to y.

   (b) Using this claim prove the following lemma:

   Suppose $x <_\sigma y$. Let t be the first vertex of the connected component of $\Gamma^\sigma_{x,y}$ containing x. Then, there exists a t, x-path P in $\Gamma^\sigma_{x,y}$ where y is not adjacent to any vertices of P with the possible exception of x. Moreover, all vertices on this path occur in σ before y.