CSC 2410F - Introduction to Graph Theory

Assignment #1 Due: Oct. 10, 2002

NB. Read the policy on "grading scheme" in the course outline. All work must be your own; reference all sources.

1. Consider the following two definitions of "similar" vertices:

   (a) x is *similar* to y iff there is an automorphism of G mapping x onto y. (Note that this is the standard definition.)

   (b) x is *similar* to y iff there is an automorphism of G interchanging x and y. (i.e. the automorphism maps x onto y and y onto x.)

   Prove or disprove that for undirected graphs, the definitions are equivalent. Note that as shown by the directed cycle on 3 vertices, the definitions are NOT equivalent for digraphs. Recall that an automorphism is an isomorphism of a graph with itself.

2. Define an n^2 node graph G_n as follows:

   To each vertex associate an ordered pair in $\{1, 2, \cdots, n\} * \{1, 2, \cdots, n\}$. Place an edge between node $< x, y >$ and node $< x', y' >$ iff $x = y'$ or $y = x'$.

   Show that G_n contains every n-node forest as an induced subgraph.

3. Show that if G and $\overline{G}$ are connected, then G (and thus $\overline{G}$ too) contains an induced copy of P_4.

   Note that it may be easier to prove the contra-positive, namely, if G has no induced P_4, then at least one of G, $\overline{G}$ is disconnected.

4. Suppose you're given a cograph G as represented by its cotree T_G.

   (a) Present a linear time algorithm that determines the number of cliques of maximum size in G.

   (b) Prove that your algorithm is correct.

   (c) Briefly justify the claim that it operates in linear time.

5. Show that the following problem is isomorphism-complete:

   Given graphs G_1 and G_2, is $G_1 \cong c(G_2)$?

   Recall that $c(G_2)$ is the centre of G_2.

   Hint: In class we saw a graph where the centre is $\overline{K_2}$. Generalize this construction.