Events in Linear-Time Properties

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A Limerick (stolen from ZUM'95)

When a program you are starting to write
To prevent it from turning out shite
Eliminate dreck
By writing a spec
And then it will all be alright

- *Ben Strulo*, BT Laboratories

Formal Specification of Systems

Would like to have a formalism that permits natural specification of systems while enabling mathematical reasoning about their correctness

Examples:

- process algebras, temporal logics, Z

Problems with adoption:

- lack of tool-support
- perceived to be difficult to apply
- considered rather expensive

Recent move to light-weight formalisms:

- use natural notations
- check only most essential properties
- apply push-button technology, whenever possible

Model-Checking

Idea: automatically check properties of finite-state systems.

Specify:

- Desired properties in temporal logic
- Model of system in some finite-state transition representation

Verifiers:

- SPIN and SMV - state-space exploration
- Concurrency Workbench - bisimulation
- COSPAN - language containment

Applications to RE:

- Verification of SCR tables
- Verification of Statecharts

New Problem

Since model-checkers can be used by fairly novice users, the language for expressing properties should be

- expressive
- easy to use

Possible candidate: Linear-time Logic (LTL)

- A language for describing and reasoning about sequences of states
- Used in mechanical analysis supported by SPIN

LTL

Comprised of

- Propositional formulas
- Temporal connectives:
  - $\Box$ (always)
  - $\Diamond$(eventually)
  - $\circ$ (next)
  - U (until)

Examples:

- $p \vee \circ q$
- $\Box(p \Rightarrow \Diamond q)$

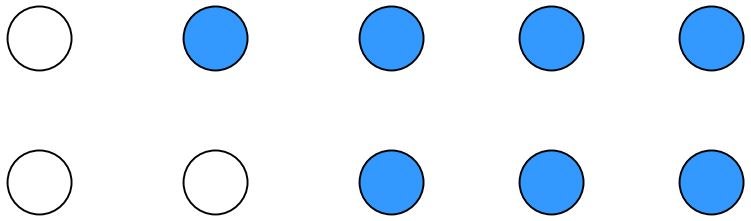
Closure Under Stuttering

Desired property of LTL formulas is *closure under stuttering* : interpretation of the formula remains the same under state sequences that differ only by repeated states [Abadi,Lamport'91].

- Guaranteed [Lamport'94] for a subset of LTL without the ○ operator

Examples:

- ⇨ $\Box a$ is closed under stuttering
- ⇨ $\bigcirc a$ is not closed under stuttering



Legend:

a is true

a is false

Notation: <<F>> - F is closed under stuttering

Using LTL to Specify Production Cell System

- Case study initiated by Forchrungszentrum Informatik (FZI)
- Aimed to show applicability of formal methods to real-world examples

Example property:

The magnet of the crane may be deactivated only when the magnet is above the feedbelt.

Resulting LTL formula:

$$\Box((activate \wedge \bigcirc\neg activate) \Rightarrow \bigcirc(head_ver = DOWN))$$

Is this formula closed under stuttering?!!

Related Work

- Determining whether an arbitrary LTL formula is closed under stutterung is PSPACE-complete [Peled,Wilke,Wolper'96]
  - ⇨ Tableu-based, $$$ approach
- A computationally-feasible algorithm for determining closure under stuttering for a subclass of formulas has been proposed [Holzmann,Kupferman'96] but not implemented in SPIN
  - ⇨ Algorithm cannot be applied by hand
  - ⇨ How useful in practice?

Our goal:

- ⇨ Want to have syntactical restrictions on LTL (like "no next state") that guarantee that the resulting formula is closed under stuttering
- ⇨ Want the approach to apply to real-life problems

Edges

$\Box((activate \wedge \bigcirc\neg activate) \Rightarrow \bigcirc(head_ver = DOWN))$

an *edge* (a change of value)

Formally, if A is an LTL formula, then

$\uparrow A = \neg A \wedge \bigcirc A$ -- up or rising edge

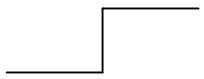
$\downarrow A = A \wedge \bigcirc\neg A$ -- down or falling edge

$\updownarrow A = \uparrow A \vee \downarrow A$ -- any edge

Example: $\uparrow\Box A$

Edges ≈ events (e.g. in SCR). Unlike SCR, can represent changes in arbitrary LTL formulas.

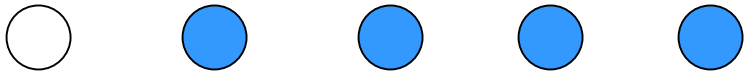
(Logical) edges ≈ signal edges



Main Result

Observation:

stuttering does not add or delete edges (or change their relative order)



Theorem:

$$<<A>> \wedge <<B>> \Rightarrow << \Diamond (\neg A \wedge \circ A \wedge \circ B) >>$$

Proof sketch: in the paper

Proof: in [Paun99]

Some Properties of Edges

- Edges are related:

$$\uparrow\neg A = \downarrow A$$

$$\downarrow\neg A = \uparrow A$$

$$\updownarrow\neg A = \updownarrow A$$

- Edges interact with each other:

$$\downarrow\downarrow A = \downarrow A$$

$$\uparrow\downarrow A = \downarrow\downarrow A$$

- Edges interact with boolean operators:

$$\uparrow(A \wedge B) = (\uparrow A \wedge \circ B) \vee (\uparrow B \wedge \circ A)$$

- Edges interact with temporal operators

$$\uparrow\circ A = \circ\uparrow A$$

$$\downarrow\Box A = \mathit{false}$$

$$\downarrow \Diamond A = \downarrow A \wedge \circ\Box\neg A$$

$$\uparrow(A\ U\ B) = \neg(A \vee B) \wedge \circ (A\ U\ B)$$

Some Properties of Closure Under Stuttering

a is a variable or a constant $\Rightarrow$ $\langle\langle a \rangle\rangle$

$$\langle\langle A \rangle\rangle = \langle\langle \neg A \rangle\rangle$$

$$\langle\langle A \rangle\rangle \wedge \langle\langle B \rangle\rangle \Rightarrow \langle\langle A \wedge B \rangle\rangle$$

$$\langle\langle A \rangle\rangle \Rightarrow \langle\langle \Box A \rangle\rangle$$

$$\langle\langle A \rangle\rangle \Rightarrow \langle\langle \Diamond A \rangle\rangle$$

$$\langle\langle A \rangle\rangle \wedge \langle\langle B \rangle\rangle \Rightarrow \langle\langle A \; U \; B \rangle\rangle$$

$$\langle\langle A \rangle\rangle \wedge \langle\langle B \rangle\rangle \Rightarrow \langle\langle A * B \rangle\rangle,$$

where $* \in \{\wedge, \vee, \Rightarrow, \Leftarrow, =\}$

Formulas of the form $\langle\langle A \rangle\rangle \Rightarrow$ f ($\uparrow A$): edges $\uparrow$ and $\downarrow$ can be used interchangeably.

Closure Under Stuttering Properties

Property 1 (Existence)

$$<<A>> \wedge <<B>> \wedge <<C>> \Rightarrow << \Diamond(\uparrow A \wedge \bigcirc B \wedge C)>>$$

with simplified versions:

$$<<A>> \wedge <<B>> \Rightarrow << \Diamond(\uparrow A \wedge B)>>$$

$$<<A>> \wedge <<B>> \Rightarrow << \Diamond(\uparrow A \wedge \bigcirc B)>>$$

Property 2 (Universality)

$$<<A>> \wedge <<B>> \wedge <<C>> \Rightarrow << \Box(\uparrow A \Rightarrow (\bigcirc B \vee C))>>$$

with simplified versions:

$$<<A>> \wedge <<B>> \Rightarrow << \Box(\uparrow A \Rightarrow B)>>$$

$$<<A>> \wedge <<B>> \Rightarrow << \Box(\uparrow A \Rightarrow \bigcirc B)>>$$

Closure Under Stuttering Properties (Cont'd)

Property 3 (Until)

$$<<A>> \wedge <<B>> \wedge <<C>> \wedge <<D>> \wedge <<E>> \wedge <<F>>$$
$$\Rightarrow << (\neg \uparrow A \vee \bigcirc B \vee C) \; U \; (\uparrow D \wedge \bigcirc E \wedge F) >>$$

with many simplified versions.

Examples:

The magnet of the crane may be deactivated only when the magnet is above the feedbelt.

$$\Box(\downarrow activate \Rightarrow \bigcirc(head_ver = DOWN))$$

Initially, no items should be dropped on the table before the operator pushes and releases the GO button

$$\neg \downarrow hold \; U \downarrow button$$

Quick Summary

- We introduced the notion of edges for LTL
- We provided a set of theorems that enable syntax-based analysis of a large class of formulas for closure under stuttering.
- Such theorems can be added to a theorem-prover for mechanized checking.

!! But the language of edges is not closed !!

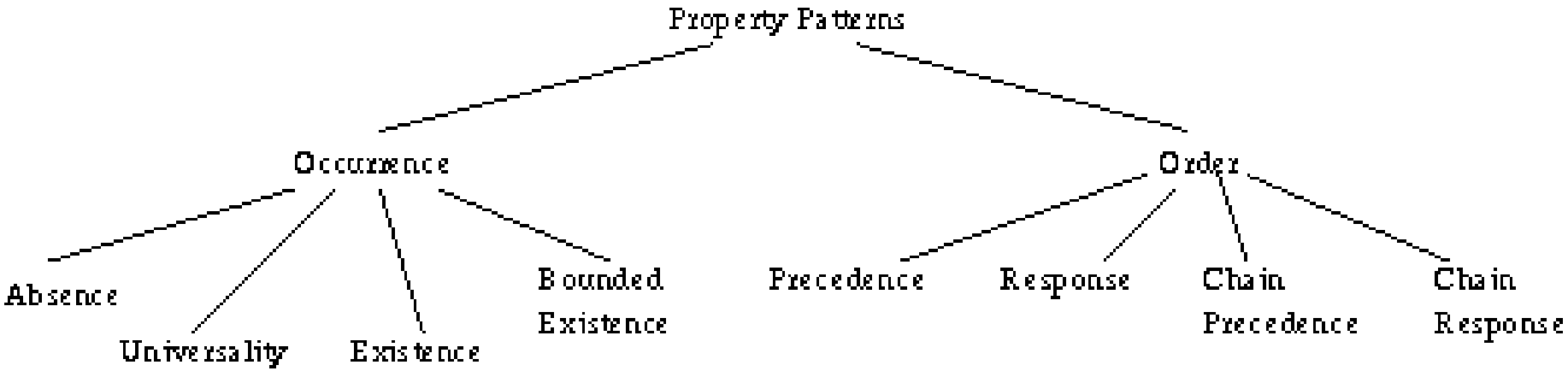
Example: $\uparrow A$

Are the properties that can be identified using our method useful in practice?

Application: Property Patterns

- Pattern-based approach [Dwyer,Avrunin,Corbett'98,'99]
  - Presentation, codification and reuse of property specifications
  - Easy conversion between formalisms: CTL, LTL, QRE, GIL…
  - Goal: to enable novice users to express complex properties effectively
    - LTL properties are state-based
- Apply our theory to
  - extend the pattern-system with events for LTL properties
  - check closure-under-stuttering of resulting formulas

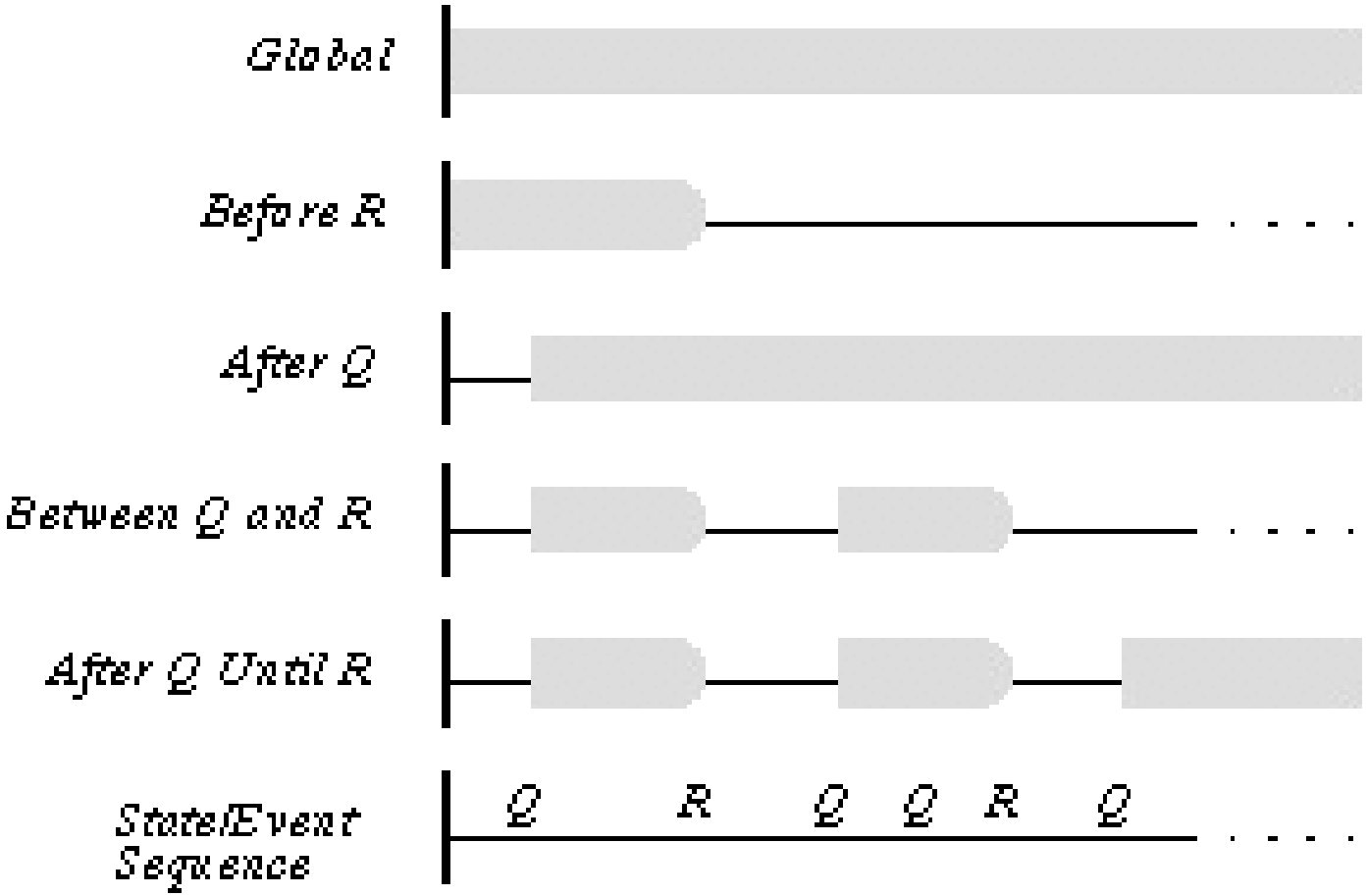
Pattern Hierarchy



- Absence — A condition does not occur within a scope
- Existence — A condition must occur within a scope
- Universality — A condition occurs throughout a scope
- Response — A condition must always be followed by another within a scope
- Precedence — A condition must always be preceded by another within a scope.

Scopes

Scopes are regions of interest over which the condition is evaluated.



Example

LTL formulation of the property

S precedes P between Q and R

(Precedence pattern with "between Q and R" scope) is

$$\Box((Q \wedge \Diamond R) \Rightarrow (\neg P \; U \; (S \vee R))))$$

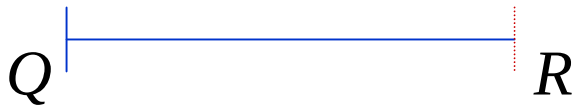
Note that S, P, Q, R are states.

Extending the Pattern System

- Want to extend LTL patterns to reasoning about events
- "next" operator: are resulting properties closed under stuttering?

Assumptions:

- ⇨ Multiple events can happen simultaneously
- ⇨ Intervals are closed-left, open-right, as in original system



Extending the Pattern System

- We have considered the following possibilities:

0.	P, S -- states	Q, R -- states
1.	P, S -- states	Q, R -- up edges
2.	P, S -- up edges	Q, R -- states
3.	P, S -- up edges	Q, R -- up edges

Note: down edges can be substituted for up edges

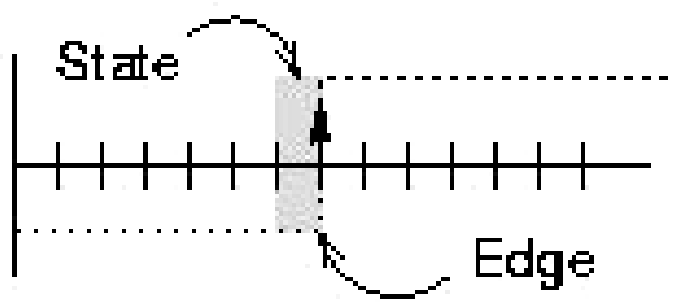
- We extended Absence, Existence, Universality, Precedence, and Response patterns.
- Some of properties from other patterns, e.g. Chain Precedence, are not closed under stuttering [paun,chechik'99]

A Note on Edges

Definition of an edge:

$$\uparrow A = \neg A \wedge \bigcirc A$$

Thus, an edge is detected in a state before it occurs.



Example: P always becomes true after Q.

Formulations:

- $\Box(Q \Rightarrow \Box P)$ if Q and P are states
- $\Box(\uparrow Q \Rightarrow \bigcirc\Box P)$ if P is a state and Q is an event

Extension of Patterns - Existence Pattern

- P Exists Before R
  0. $\Diamond R \Rightarrow \neg(\neg P \; U \; R)$
  1. $\Diamond \uparrow R \Rightarrow (\neg \uparrow R \; U \; P)$
  2. $\Diamond R \Rightarrow \neg(\neg \uparrow P \; U \; R)$
  3. $\Diamond \uparrow R \Rightarrow \neg(\neg \uparrow P \; U \uparrow R)$
- P Exists Between Q and R
  0. $\Box(Q \wedge \Diamond R \Rightarrow \neg(\neg P \; U \; R) \wedge \neg R)$
  1. $\Box(\uparrow Q \wedge \Diamond \uparrow R \Rightarrow \bigcirc(\neg \uparrow R \; U \; P) \wedge \neg \uparrow R)$
  2. $\Box(Q \wedge \Diamond R \Rightarrow \neg(\neg \uparrow P \; U \; R) \wedge \neg R)$
  3. $\Box(\uparrow Q \wedge \Diamond \uparrow R \Rightarrow \neg(\neg \uparrow P \; U \uparrow R) \wedge \neg \uparrow R)$

Using the Pattern System: Example

Example property:

The robot must weigh the blank after pickup from the feedbelt, but before depositing it on the press.

Variables:

(state) mgn - true when the magnet is on

(state) scl - the scale reports a successful weighing

This is the Existence pattern: weighing (state) must happen between (events) pickup and deposit. Scope is Between R and Q.

Pattern Formula:

$$\Box(\uparrow Q \wedge \Diamond \uparrow R \Rightarrow \bigcirc(\neg \uparrow R \; U \; P) \wedge \neg \uparrow R)$$

Resulting Formula:

$$\Box(\uparrow mgn \wedge \Diamond \downarrow mgn \Rightarrow \bigcirc(\neg \downarrow mgn \; U \; scl) \wedge \neg \downarrow mgn)$$

Proving Closure Under Stuttering

- Can use properties of closure under stuttering, the algebra of edges, and rules of logic to show

  $$(<<P>> \wedge <<Q>> \wedge <<R>>) \Rightarrow$$
  $$<<\Box(\uparrow Q \wedge \Diamond\uparrow R \Rightarrow \bigcirc(\neg\uparrow R \; U \; P) \wedge \neg\uparrow R)>>$$

  in roughly 8 steps (see paper) completely syntactically.

- We proved all new edge-based formulas for closure under stuttering.
- Users can use these without worrying

Summary of the Problem

- The "next" operator in LTL is required for reasoning about events
- " "next" is present => the result is not closed under stuttering"
- Solution: introduce extra variables to simulate events:
  - ⇨ Clutter the model, make harder to analyze
  - ❗ Results of verification cannot be interpreted correctly, without complete understanding of the modeling language and LTL. So, novice users will be making mistakes!!!

Summary of Solution

- We introduced the notion of edges for LTL
- We provided a set of theorems that enable syntax-based analysis of a large class of formulas for closure under stuttering.
- Such theorems can be added to a theorem-prover for mechanized checking.
- The language is not closed (unlike "next"-free LTL)
- But it can express properties useful in practice:
  - Properties of Production Cell [Paun,Chechik,Biechele'98]
  - Property patterns + events [Paun,Chechik'99]
- For more information:
  `http://www.cs.toronto.edu/~chechik/edges.html`

Please send us examples of the use of edges!!!!!