Function Symbols in Prolog

In logic, there are two kinds of objects: predicates and functions.

- Predicates represent statements about the world:
  `John hates Mary: hates(john,mary).`
  `John is short: short(john)`

(`hates` is a predicate symbol, `short(john)` is an atomic formula)

- Function terms represent objects in the world
  `the mother of Mary: mother-of(mary)`
  `a rectangle of length 3 and width 4: rectangle(3,4)`

(`mother-of(mary)` is a function term, `rectangle` is a function symbol)

Function terms do not have values. In Prolog, the act like data structures:
let `p2(X,Y)` denote a point in 2-dim space
let `p3(X,Y,Z)` denote a point in 3-dim space.
Write a Prolog program, `SQDIST(Point1,Point2,D)`, that returns the square of the distance between two points. The program should work for 2- and 3-dim points.

Want:

```
SQDIST(p2(1,2), p2(3,5), D)
  returns D = (3-1)**2 + (5-2)**2
            = 4+9 = 13
and
SQDIST(p3(1,1,0), p3(2,2,3), D)
  returns D = (1-2)**2 + (1-2)**2 + (0-3)**2
            = 1+1+9=11
and
SQDIST(p2(0,0), p3(1,1,1), D)
  is undefined
```


The Prolog program:

```
(1) SQDIST(p2(X1,Y1), p2(X2,Y2), D)
      :- XD is X1-X2,
         YD is Y1-Y2,
         D is XD*XD + YD*YD.
(2) SQDIST(p3(X1,Y1,Z1), p3(X2,Y2,Z2), D)
       :- XD is X1-X2,
          YD is Y1 - Y2,
          ZD is Z1 - Z2,
          D is XD * XD + YD*YD + ZD*ZD.
```

Query: `SQDIST(p2(1,2), p2(3,5), D)`
This query unifies with the head of rule `(1)` with `{X1\1, Y1\2, X2\3, Y2\5}`

so, `XD` is `X1-X2=1-3=-2`
`YD` is `Y1-Y2=2-5=-3`
`D` is $(-2)^2 + (-3)^2 = 13$
So, `D=13` is returned

Note: the query does not unify with the head of rule `(2)`, so only rule `(1)` is used.

Query: `SQDIST(p3(1,1,0),p3(2,2,3),D).`
The query unifies with the head of rule `(2)`, with `{X1\1, Y1\1, Z1\0, X2\2, Y2\2, Z2\3}`

so, `XD` is `1-2 = -1`
`YD` is `1-2 = -1`
`ZD` is `0-3 = -3`
`D` is `1 + 1 + 9 = 11`
So, `D=11` is returned

Note: this query does not unify with the head of rule `(1)`, so only rule `(2)` is used.

Query: `SQDIST(p2(0,0), p3(1,1,1), D).`
The query does not unify with any rule, so Prolog simply returns `no`, i.e., no answers for `D`.

Returning Function Terms as Answers

Eg. Given a point, `p2(X,Y)`, return a new point with double the coordinates.

Query `double(p2(3,4),P)`
Answer `P = p2(6,8).`

Prolog program:

```
double(p2(X1,Y1), p2(X2,Y2))
   :- X2 is 2*X1,
      Y2 is 2*Y1.
```

i.e., if `x2 = 2 * X1` and `Y2 = 2 * Y1`, then the double of `p2(X1,Y1)` is `p2(X2,Y2)`.

An equivalent program using "="

```
double(p2(X1,Y1), P)
   :- X2 is 2*X1, Y2 is 2*Y1,
      P = p2(X2,Y2).
```

Here, "=" is being used to assign a value to variable `P`. Try to avoid this!!!!! It reflects procedural thinking.

Returning Terms as Answers (Cont'd)

The query `double(p2(3,4),P)` unifies with the head of the rule with

`mgu = {X1\3, Y1\4, P\p2(X2,Y2)}.`

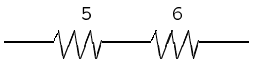
The body of the rule then evaluates:

`X2 is 2*X1`, i.e., 6
`Y2 is 2*Y2`, i.e., 8

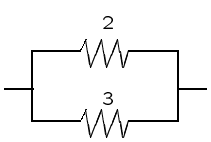
The mgu becomes `{X1\3, Y1\4, P\p2(6,8)}`, so the answer is `P = p2(6,8).`

Recursion with Function Symbols

Example: Electrical circuits

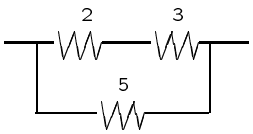


- Two resistors in series, with resistances R_1 and R_2, respectively.
- Total resistance of the circuit is $5+6=11$.
- Can represent the circuit as a function term: `series(5,6).`

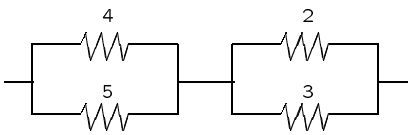


- Two resistors in parallel.
- Total resistance of the circuit is $\frac{2+3}{2\times 3}=1.2$
- Represent the circuit as a function term: `par(2,3).`

More Complex Circuits



`par(5, series(2,3))`



`series(par(4,5), par(2,3))`

Circuits (Cont'd)

Problem: Write a Prolog program that computes the total resistance of any circuit. E.g.

Query: `resistance(series(1,2), R)`
Answer: `R = 1+2 = 3`

Query: `resistance(par(2,3), R)`
Answer: `R = (2*3)/(2+3) = 6/5 = 1.2`

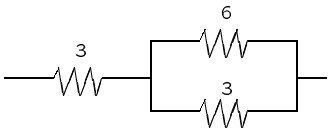
Query: `resistance(series(3,par(2,3)), R)`
Answer: `R = 3 + 1.2 = 4.2`

Query: `resistance(3, R)`
Answer: `R = 3`

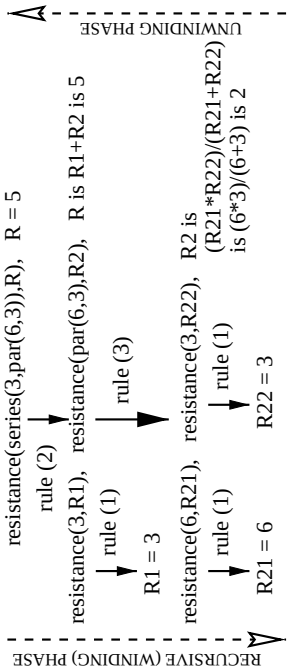
Solution:

```
(1) resistance(R,R) :- number(R).
(2) resistance(series(C1,C2), R)
       :- resistance(C1, R1),
          resistance(C2, R2),
          R is R1 + R2.
(3) resistance(par(C1,C2), R)
       :- resistance(C1,R1),
          resistance(C2,R2),
          R is (R1 * R2) / (R1 + R2).
```


Example



Query: `resistance(series(3,par(6,3)), TR)`



Lists in Prolog

special constant symbol	[ ]	≈	`nil`
special function symbol	•	≈	`cons`

Lisp list: `(a b c)`
Prolog list: `•(a, •(b, •(c, [])))`
≈ `(cons 'a (cons 'b (cons 'c nil)))`

Prolog shorthand:

1. $[t_1, \ldots, t_n] \equiv$ (`list` $'t_1 \cdots 't_n$)

2. $[t_1, \ldots, t_n \mid \mathtt{L}] \equiv$ (`cons` $'t_1$ (`cons` $'t_2$ ($\cdots$ (`cons` $'t_n$ `L`) $\cdots$)

Thus `[a,b,c]` ≡ `[a | [b,c]]`
≡ `[a,b | [c]]`
≡ `[a,b,c | []]`

List Membership

Prolog uses predicates, not functions. Want to define a two-place predicate

`member(X,Y)`

that holds iff `X` is a member of list `Y`.

Solution:

```
(1) member(X,[X|Y]).
(2) member(X,[Z|Y]) :- member(X,Y).
```

Interpretation:

1. `X` is a member of any list of which it is the first element.
2. If `X` is a member of list `Y`, then `X` is a member of any list that has `Y` as its `cdr`.

An Example

```
| ?- trace, member(c,[a,b,c,d]).
   (2) 1 Call: member(c,[a,b,c,d]) ?
   (3) 2 Call: member(c,[b,c,d]) ?
   (4) 3 Call: member(c,[c,d]) ?
   (4) 3 Exit: member(c,[c,d])
   (3) 2 Exit: member(c,[b,c,d])
   (2) 1 Exit: member(c,[a,b,c,d])
yes
| ?- trace, member(x,[a,b,c]).
   (2) 1 Call: member(x,[a,b,c]) ?
   (3) 2 Call: member(x,[b,c]) ?
   (4) 3 Call: member(x,[c]) ?
   (5) 4 Call: member(x,[]) ?
   (5) 4 Fail: member(x,[])
   (4) 3 Fail: member(x,[c])
   (3) 2 Fail: member(x,[b,c])
   (2) 1 Fail: member(x,[a,b,c])
no
```


Inductive Reasoning

- Identify Base Cases
  Handled directly as facts
  Simple solution, e.g null

- Identify Decomposition
  Reduce General Case to simpler sub cases

- Identify Composition
  Compose solution of Subcases to solution of original general case

Example - member

- Desired, `member( X , Y )`
  succeeds if X is a member of list Y
- Base case:
  X is at head of list Y
  `member( X , [X | Y] ).`
  (List Y is empty
  `member(X, []) :- fail.`)
- Induction case:
  X is member of rest of Y except head
  `member( X, [Z|Y] ) :- member( X, Y).`
- Order of axioms matters.

Example - Append

- Desired: `append(X,Y,Z)`
  Z is result of appending X onto Y
- Base case:
  Append empty list onto Y
  `append( [], Y, Y ).`
- Decomposition:
  Process head of X
  Process rest of X
  Reduce length of X
- Composition:
  Add to head of Z
- Solution:
  `append([E|X],Y,[E|Z]):-append(X,Y,Z).`

An Example

append([a,b],[c,d],V)

(2) $E_1 = a \quad X_1 = [b]$
$Y_1 = [c,d] V = [a|Z_1]$

append([b],[c,d],Z_1)

(2) $E_2 = b \quad X_2 = []$
$Y_2 = [c,d] Z_1 = [b|Z_2]$

append([ ],[c,d],Z_2)

(1) $Y_2 = [c,d]$
$Z_2 = [c,d]$

success

`V` $= [a\ |Z_1]$ $= [a \mid [b\ |Z_2]]$
$= [a \mid [b \mid [c,d]]]$ $= [a,b,c,d]$

More Examples

append([a,b],[c],[a,b,c,d])

(2) $E_1 = a \quad X_1 = [b]$
$Y_1 = [c] \quad Z_1 = [b,c,d]$

append([b],[c],[b,c,d])

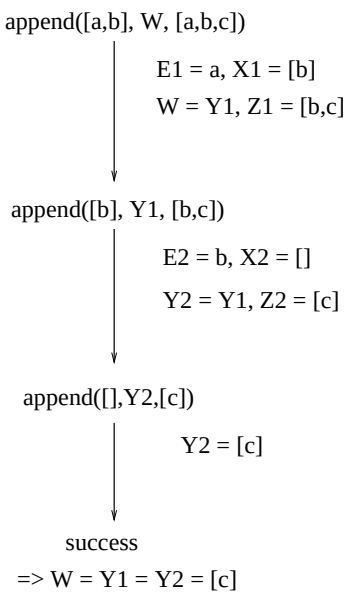
(2) $E_2 = b \quad X_2 = []$
$Y_2 = [c] \quad Z_2 = [c,d]$

append([ ],[c],[c,d])

No possible unification.

failure

More Examples



Example - Reverse a List

- Desired `rev( L , M )`
  where the list M is the result of reversing the order of the list L
- Solution without helper functions
- Base case:
  Reverse empty list
  `rev( [], [] ).`
- Induction case:
  Append head of L to tail of output
  `rev([H|T],M) :- rev(T,Z),append(Z,[H],M).`

Example - Reverse a List with Helper Function

- Use helper function to hold partial result
- Base case:
  Start with empty output list
  `rev2(L1,L2) :- revzap(L1,[],L2).`
- Induction case:
  Accumulate results so far in 2nd arg of revzap
  `revzap([],L,L).`
  `revzap([X | L],L2,L3):-revzap(L,[X | L2],L3).`

Equality in Lists

The predicate for equality of lists is trivial:

```
equal(X,X).
```

Unification does all the work!

Note that `equal(`t_1`,`t_2`)` succeeds if t_1 and t_2 are merely unifyable:

`equal([a,b,c],[a|X])` $\leadsto$ success + `X = [b,c]`

An Extended Example

We will represent the natural numbers as Prolog terms:

constant		0
unary function symbol		`s` ≈ `successor`

0	represents	0
s(0)	...	1
s(s(0))	...	2
s(s(s(0)))	...	3
	etc.	

[Prolog does have numbers, but we ignore them for now.]

Addition Predicate

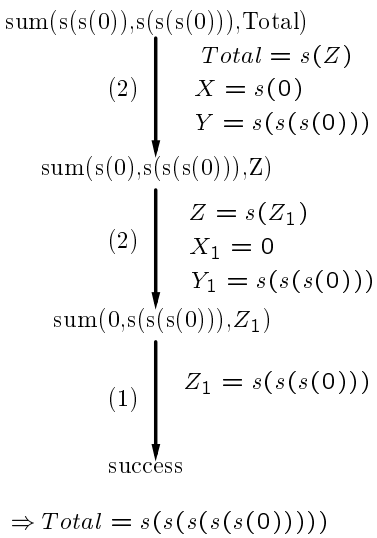
Want to define `sum`:

`sum(X,Y,Z)` holds iff `X` + `Y` = `Z`.

Solution:

```
sum(0,Y,Y).                          (1)
sum(s(X),Y,s(Z)) :- sum(X,Y,Z)       (2)
```

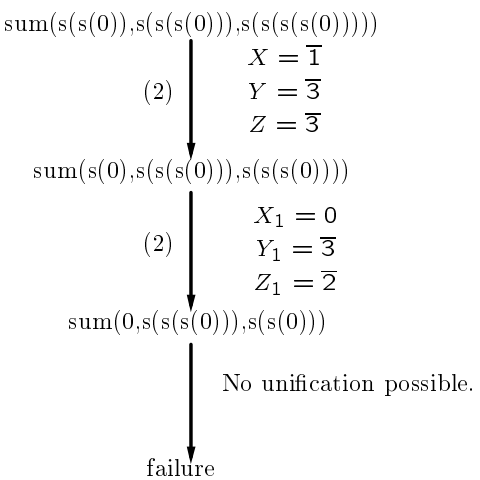

An Example



$\Rightarrow Total = s(s(s(s(s(0)))))$

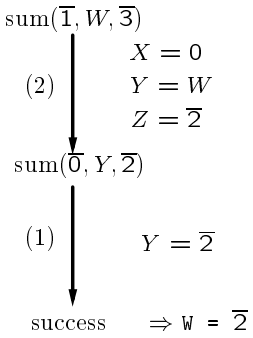
Another Example

Notation: $\overline{n} \equiv \underbrace{s(s(\cdots s(}_{n \text{ times}} 0)\cdots)$



$\Rightarrow$ `sum`$(\overline{2}, \overline{3}, \overline{4})$ fails

Addition as Subtraction



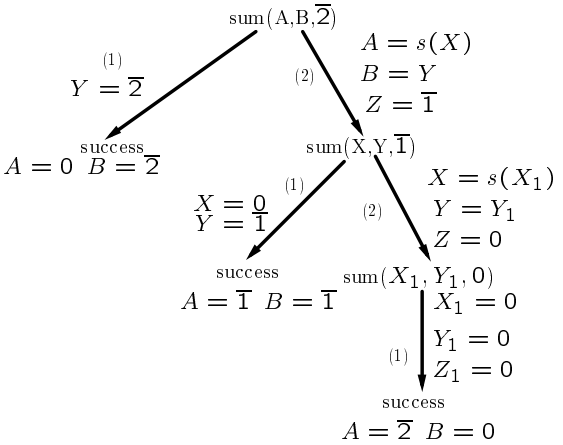
Similarly, `sum(D,`$\overline{3}$`,`$\overline{8}$`)` $\rightsquigarrow$ `D` $= \overline{5}$

$\Rightarrow$ can define subtraction via one axiom:

```
diff(X,Y,Z) :- sum(Y,Z,X).
```

`diff(X,Y,Z)` holds iff $X - Y = Z$.

Creative Use of Variables



Similarly, `sum(A,B,`$\overline{5}$`)` succeeds with

$A = 0$	$B = \overline{5}$
$A = \overline{1}$	$B = \overline{4}$
$A = \overline{2}$	$B = \overline{3}$

etc.

A Bigger Example: Sorting

Declarative approach:

```
sort(X,Y) :- permute(X,Y), ordered(Y).
```

i.e. `Y` is the sorted version of `X` if `Y` is a permutation of `X` and `Y` is ordered.

`permute` and `ordered` are not difficult to define, but `ordered` will depend of the type list element.

However, this is a terrible way to sort.

A list with n elements has $n!$ permutations, and each of them will be tried until one is found to be ordered.

Sorting Lists

- Common Operation for Organizing Data
- Could use builtin list sorting primitives in Prolog. We'll look at how to do it directly.
- Assume that there is an ordering relation:
  `gt(X,Y)`
  for whatever data type we want to sort.
  Also assume non-empty lists.
- Look at several alternatives:
  bubble sort
  insertion sort
  quick sort

Basic Bubble Sort

Algorithm: swap adjacent elements in the list until all items are in order

```
bubblesort( List, Sorted ) :-
    swap( List, List1 ),
    bubblesort( List1, Sorted ).

bubblesort( Sorted, Sorted ).

swap( [] , [] ).
swap( [ X, Y | Rest ], [ Y, X | Rest ]) :-
    gt( X, Y ).
swap( [ Z | Rest ] , [ Z | Rest1 ] ) :-
    swap( Rest, Rest1 ).
```


Insertion Sort

- To sort a non-empty list `L = [X|T]`
  - Sort the tail T of L
  - Insert the head X into the sorted tail such that the resulting list is sorted

```
insertsort( [], [] ).
insertsort([ X | Tail ], Sorted ) :-
    insertsort( Tail, SortedTail ),
    insert( X, SortedTail, Sorted ).
insert( X, [ Y, Sorted ], [ Y | Sorted1 ] ) :-
    gt( X, Y ) , ! ,
    insert( X, Sorted, Sorted1 ).
insert( X, Sorted, [ X | Sorted ] ).
```


Quicksort

To sort a non-empty list L:

- Select some element X from L as a *pivot*
- Split the rest of L into two lists (Small and Big) such that all elements of L greater than X are in Big and all the rest are in Small
- Sort Small to obtain SortedSmall
  Sort Big to obtain SortedBig
- The whole sorted list is the concatenation of
  SortedSmall and `[ X | SortedBig ]`

Quicksort Algorithm

```
quicksort([] , [] ).
quicksort([X | Tail ], Sorted) :-
    split(X, Tail, Small, Big),
    quicksort(Small, SortedSmall),
    quicksort(Big, SortedBig),
    conc(SortedSmall,[ X | SortedBig],Sorted).

split(X, [], [], []).
split(X, [ Y | Tail ], [ Y | Small ], Big):-
    gt( X, Y ), ! ,
    split( X, Tail, Small, Big ).
split(X, [ Y | Tail ], Small, [ Y | Big ]) :-
    split( X, Tail, Small, Big ).

conc( [], L, L ).
conc( [ X | L1 ], L2, [ X | L3 ] ):-
    conc( L1, L2, L3 ).
```

More Efficient List Concatenation

- Naive list concatenation:

```
conc( [], L, L ).
conc( [ X | L1 ], L2, [ X | L3 ] ):-
    conc( L1, L2, L3 ).
```

- This is very inefficient when first list is long.
  Recurs down first list until it is empty.
  Could do better if we could efficiently find end of first list.

- Change representation of lists to represent a list as a pair of lists.
  A *Difference List* represents a list L and a difference of two lists L1 and L2, written as L1-L2

Difference List Example

```
Let   L = [ a, b, c ]

Then L can be represented as L1-L2 using any of

      L1 = [ a, b, c, d, e ]
      L2 = [ d, e ]

      L1 = [ a, b, c ]
      L2 = [ ]

      L1 = [ a, b, c, d, e | T ]
      L2 = [ d, e | T ]

Where T is any list.
Note that L2 is the end of the list L.
```


Concatenation of Difference Lists

```
Difference list concatenation:

concat( A1-Z1, Z1-Z2, A1-Z2 ).

Example:
Let
    M = [ a, b, c ]
    M = [ a, b, c | T1 ] - T1
    N = [ d, e ]
    N = [ d, e | T2 ] - T2
Then
   concat( M, N )
is
   concat([a, b, c | T1]-T1,[d, e, T2]-T2,L)
Resulting in
   T1 = [ d, e, | T2 ]
   L  = [ a, b, c, d, e | T2 ] - T2
```


More Efficient Quicksort Using Difference Lists

```
quicksort( List, Sorted ) :-
   quicksort2(List,Sorted-[]).

quicksort2([],Z-Z).

quicksort2([ X | Tail], A1-Z2) :-
    split(X, Tail, Small, Big ),
    quicksort2(Small, A1-[X | A2]),
    quicksort2(Big, A2-Z2 ).
```

Prolog - Review

- Overview
- Predicate Calculus
- Substitution and Unification
- Introduction to Prolog
- Prolog Inference Rules
- Programming in Prolog
  - Recursion
  - List Processing
  - Arithmetic
  - Higher-order programming
- Conclusion

Prolog vs. "Standard" Procedures

1. Success vs. failure

```
p(X) :- <body 1>.
p(X) :- <body 2>.
```

Procedural: Try `<body 1>`.
If that fails, try `<body 2>`.

2. Automatic backtracking

```
q(a,b).
q(a,c).
q(a,d).
r(c).
r(d).
p(X) :- q(X,Y), r(Y).
```

Call `p(a)`. Assign $Y = b$ then fails.
Reassign $Y = c$ then succeeds.
Can tell Prolog to find another proof.
Reassign $Y = d$ then succeeds.
Try again $\leadsto$ failure.

3. Non-determinism (in principle)
We can execute a goal in more than one way.

```
p(X) :- <body 1>.
p(X) :- <body 2>.
```

Which body will be executed?
(Prolog's use of DFS makes this deterministic.)

4. Variable use

- variables can be given values before proc. call or by proc. call
- variables need not have or be given values at all, but can simply be constrained with respect to other variables via unification.
- can defer decisions about particular values that variables must have as long as possible
  $\longrightarrow$ major strength of Prolog.

food for slide eater