SAT and Model Checking

Bounded Model Checking (BMC)

Biere, Cimatti, Clarke, Zhu, 1999

- A.I. Planning problems: *can we reach a desired state in k steps?*
- Verification of *safety* properties: *can we find a bad state in k steps?*
- Verification: *can we find a counterexample in k steps ?*

What is SAT?

Given a propositional formula in CNF, find if there exists an assignment to Boolean variables that makes the formula true:

literals

clauses

$\omega_1 = (b \vee c)$

$\omega_2 = (\neg a \vee \neg d)$

$\omega_3 = (\neg b \vee d)$

$\varphi = \omega_1 \wedge \omega_2 \wedge \omega_3$

$A = \{a=0, b=1, c=0, d=1\}$

SATisfying assignment!

BMC idea

Given: transition system *M*, temporal logic formula *f*, and user-supplied time bound *k*

Construct propositional formula Ω(k) that is *satisfiable* iff *f* is valid along a path of length *k*

Path of length *k*: $I(s_0) \wedge \bigwedge_{i=0}^{k-1} R(s_i, s_{i+1})$

Say f = **EF** p and $k = 2$, then

$$\Omega(2) = I(s_0) \wedge R(s_0, s_1) \wedge R(s_1, s_2) \wedge (p_0 \vee p_1 \vee p_2)$$

What if f = **AG** p ?

BMC idea (cont'd)

AG *p* means *p* must hold in every state along any path of length *k*

We take

$$\neg\Omega(k) = (I(s_0) \wedge \bigwedge_{i=0}^{k-1} R(s_{i,}s_{i+1})) \rightarrow \bigwedge_{i=0}^{k} p_i$$

So

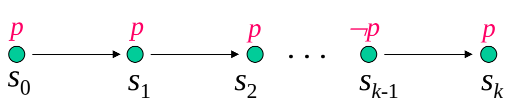
$$\Omega(k) = I(s_0) \wedge \bigwedge_{i=0}^{k-1} R(s_{i,}s_{i+1}) \wedge \bigvee_{i=0}^{k} \neg p_i$$

That means we look for counterexamples

Safety-checking as BMC

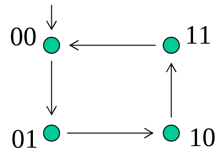
p is preserved up to *k-th* transition iff $\Omega(k)$ is unsatisfiable:

$$\Omega(k) = I(s_0) \wedge \bigwedge_{i=0}^{k-1} R(s_{i,}s_{i+1}) \wedge \bigvee_{i=0}^{k} \neg p$$



If satisfiable, satisfying assignment gives counterexample to the safety property.

Example: a two bit counter



Initial state: $I : \neg l \wedge \neg r$

Transition: $R : \begin{pmatrix} l' = (l \neq r) \wedge \\ r' = \neg r \end{pmatrix}$

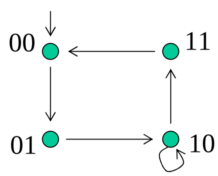
Safety property: **AG** $(\neg l \vee \neg r)$

$$\Omega(2) : (\neg l_0 \wedge \neg r_0) \wedge \begin{pmatrix} l_1 = (l_0 \neq r_0) \wedge r_1 = \neg r_0 \wedge \\ l_2 = (l_1 \neq r_1) \wedge r_2 = \neg r_1 \end{pmatrix} \wedge \begin{pmatrix} (l_0 \wedge r_0) \vee \\ (l_1 \wedge r_1) \vee \\ (l_2 \wedge r_2) \end{pmatrix}$$

$\Omega(2)$ is unsatisfiable. $\Omega(3)$ is satisfiable.

Example: another counter

$I : \neg l \wedge \neg r \qquad R : \begin{pmatrix} l' = (l \neq r) \wedge \\ r' = \neg r \end{pmatrix} \vee \begin{pmatrix} l' = l \wedge \\ r' = r \wedge \\ l \wedge \neg r \end{pmatrix}$



Liveness property: **AF** $(l \wedge r)$

Check: **EG** $(\neg l \vee \neg r)$

$$\Omega(2) = I(s_0) \wedge \bigwedge_{i=0}^{1} R(s_i, s_{i+1}) \wedge \bigvee_{i=0}^{2} (\neg l_i \vee \neg r_i) \wedge loop$$

where

$$loop = R(s_2, s_3) \wedge (s_3 = s_0 \vee s_3 = s_1 \vee s_3 = s_2)$$

$\Omega(2)$ is satisfiable

Satisfying assignment gives counterexample to the liveness property

What BMC with SAT Can Do

- All LTL
- ACTL and ECTL
- In principle, all CTL and even mu-calculus
  - efficient universal quantifier elimination or fixpoint computation is an active area of research

How big should *k* be?

- For every model *M* and LTL property φ there exists *k* s.t.

$$M \models_k \varphi \rightarrow M \models \varphi$$

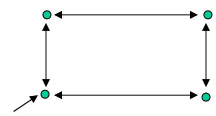
- The minimal such *k* is the *Completeness Threshold* (*CT*)

How big should *k* be?

- *Diameter* *d* = longest shortest path from an initial state to any other reachable state.
- *Recurrence Diameter* *rd* = longest loop-free path.
- $rd \geq d$

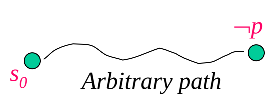
d = 2

rd = 3



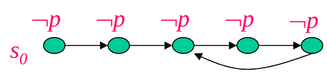
How big should *k* be?

- *Theorem*: for **G***p* properties $CT = d$



How big should *k* be?

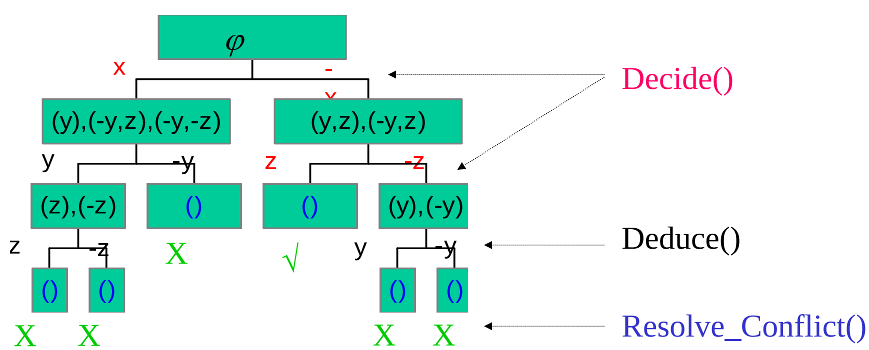
- *Theorem*: for **F***p* properties *CT= rd*



◆ Open Problem: The value of *CT* for general Linear Temporal Logic properties is unknown

A basic SAT solver

Given φ in CNF: (x,y,z),(-x,y),(-y,z),(-x,-y,-z)



Basic Algorithm

```
While (true)
{
        if (!Decide()) return (SAT);
        while (!Deduce())
                if (!Resolve_Conflict()) return (UNSAT);
}
```

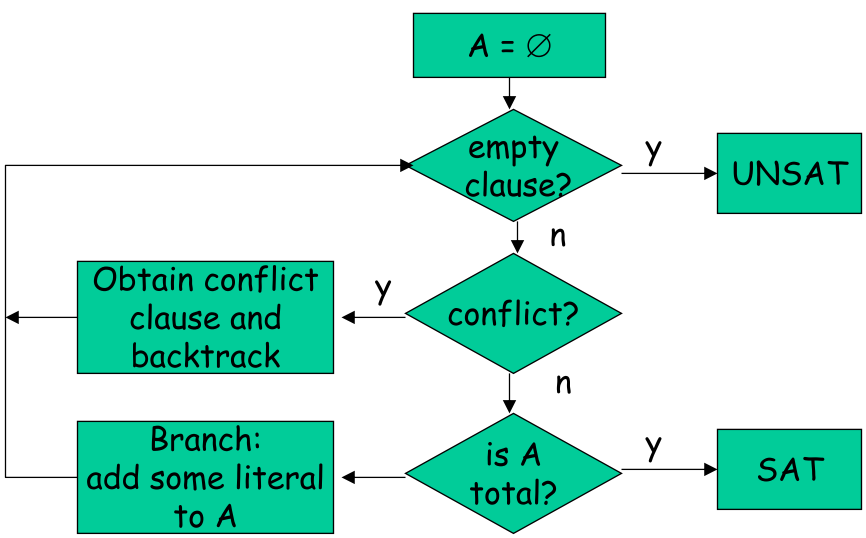
Choose the next variable and value. Return False if all variables are assigned

Apply unit clause rule. Return False if reached a conflict

Backtrack until no conflict. Return False if impossible

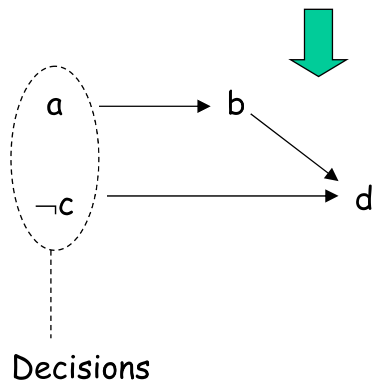
DPLL-style SAT solvers

SATO,GRASP,CHAFF,BERKMIN



The Implication Graph

$(\neg a \vee b) \wedge (\neg b \vee c \vee d)$



Assignment: $a \wedge b \wedge \neg c \wedge d$

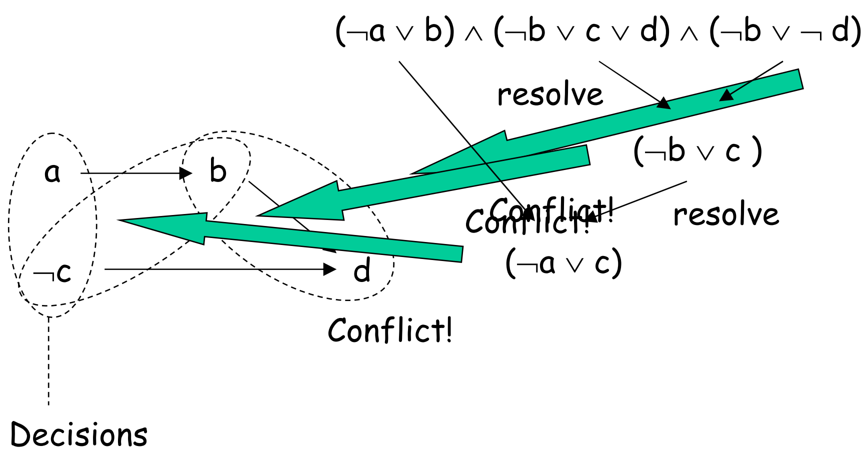
Resolution

$a \vee b \vee \neg c$ $\qquad\qquad$ $\neg a \vee \neg c \vee d$

$b \vee \neg c \vee d$

When a conflict occurs, the implication graph is used to guide the resolution of clauses, so that the same conflict will not occur again.

Conflict clauses



Assignment: $a \wedge b \wedge \neg c \wedge d$

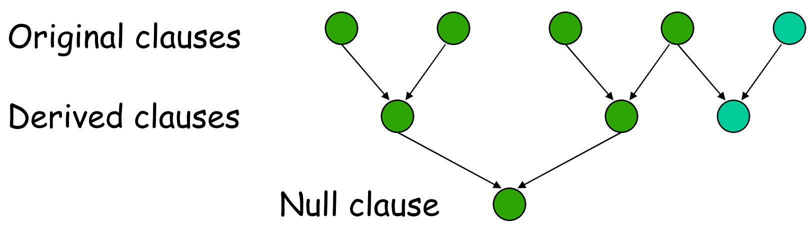
Conflict Clauses (cont.)

- Conflict clauses:
  - Are generated by resolution
  - Are implied by existing clauses
  - Are in conflict with the current assignment
  - Are safely added to the clause set

Many heuristics are available for determining when to terminate the resolution process.

Generating refutations

- Refutation = a proof of the null clause
  - Record a DAG containing all resolution steps performed during conflict clause generation.
  - When null clause is generated, we can extract a proof of the null clause as a resolution DAG.



Unbounded Model Checking

- A variety of methods to exploit SAT and BMC for unbounded model checking:
  - Completeness Threshold
  - *k* - induction
  - Abstraction (refutation proofs useful here)
  - Exact and over-approximate image computations (refutation proofs useful here)
  - Use of Craig approximation

Conclusions: BDDs vs. SAT

- Many models that cannot be solved by BDD symbolic model checkers, can be solved with an optimized SAT Bounded Model Checker.
- The reverse is true as well.
- BMC with SAT is faster at finding shallow errors and giving short counterexamples.
- BDD-based procedures are better at proving absence of errors.

Acknowledgements

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