CSC 2125, Fall 2006
Homework 2 Solutions

October 3, 2006

1. We start with $(\langle s_p, d_1\rangle \to \langle n, d_2\rangle) = (\langle s_p, g\rangle \to \langle e_p, g\rangle)$. Since $n = e_p$, we match the case on line [21].

   Now, $callers(p) = \{n_2, n_7\}$. Since the order is unspecified on line [22], we'll process n_2 first.

   Line [23] has us search for calls to procedure p from n_2 and returns from p into $returnSite(n_2)$ associated with g. $returnSite(n_2) = n_3$. Since $\langle n_2, g\rangle \to \langle s_p, g\rangle$ and $\langle e_p, g\rangle \to \langle n_3, g\rangle$ are the only ones that match (consulting $E^{\#}$ in Figure 2), we get $d_4 = g$ and $d_5 = g$ on line [23].

   By assumption, the check on line [24] passes, and on line [25], we add the new summary edge $\langle n_2, g\rangle \to \langle n_3, g\rangle$. Line [26] then asks us for d_3's such that $\langle s_{main}, d_3\rangle \to \langle n_2, g\rangle$. Consulting Figure 2 again, the only candidate is $d_3 = \mathbf{0}$, so we may call `Propagate()` on $\langle s_{main}, d_3\rangle \to \langle n_3, g\rangle$. Looking at Figure 2, we see that $\langle n_3, g\rangle$ is indeed labelled as reachable.

   Now consider $c = n_7$. $returnSite(n_7) = n_8$, and we get $d_4 = d_5 = g$. We thus add $\langle n_7, g\rangle \to \langle n_8, g\rangle$ to our set of summary edges.

   There is no d_3 such that $\langle s_p, d_3\rangle \to \langle n_7, g\rangle$; hence, we don't call `Propagate()` at all. This is consistent with Figure 2: although there is an edge in $E^{\#}$ from $\langle e_p, g\rangle$, which is reachable, to $\langle n_8, g\rangle$, which is not, the "possibly-uninitializedness" is not propagated along it.

2. The representation function for $f(X) = D - X$ is

   $$\{(\mathbf{0}, \mathbf{0}), (\mathbf{0}, a), (\mathbf{0}, b), (\mathbf{0}, c)\}$$

   $[[R_f]] \neq f$. To see this, note that $f(\{a\}) = \{b, c\}$, but $[[R_f]](\{a\}) = \{a, b, c\}$.

3. The proof is given in the RHS tech report at `ftp://ftp.diku.dk/diku/semantics/papers/D-215.ps.Z`. Let D be given, and assume that f distributes over $\cup$. Let $X \subseteq D$. Then,

$$
\begin{array}{lll}
 & f(X) & \\
= & \bigcup_{x\in X} f(\{x\}) \cup f(\emptyset) & \text{since } f \text{ is distributive} \\
= & \bigcup_{x\in X} \{y|y \in f(\{x\})\} \cup \{y|y \in f(\emptyset)\} & \\
= & \bigcup_{x\in X} \{y|y \in f(\{x\}) \vee y \in f(\emptyset)\} & \\
= & \{y|(\exists x \in X \cdot y \in f(\{x\})) \vee y \in f(\emptyset)\} &
\end{array}
$$

Now, using the above, we write,

$$
\begin{array}{llr}
 & [[R_f]](X) & \\
= & \{y|\exists x \in X \cdot (x,y) \in R_f\} \cup \{y|(\mathbf{0},y) \in R_f\} - \{\mathbf{0}\} & \text{definition of } [[R_f]] \\
= & \{y|\exists x \in X \cdot y \in f(\{x\}) \wedge y \notin f(\emptyset)\} \cup \{y|(\mathbf{0},y) \in R_f\} - \{\mathbf{0}\} & \text{part of definition of } R_f \\
= & \{y|\exists x \in X \cdot y \in f(\{x\}) \wedge y \notin f(\emptyset)\} \cup \{y|y \in f(\emptyset) \vee y = \mathbf{0}\} - \{\mathbf{0}\} & \text{other part of definition of } R_f \\
= & \{y|\exists x \in X \cdot y \in f(\{x\}) \wedge y \notin f(\emptyset)\} \cup \{y|y \in f(\emptyset)\} \cup \{\mathbf{0}\} - \{\mathbf{0}\} & \text{re-arrange} \\
= & \{y|\exists x \in X \cdot y \in f(\{x\}) \wedge y \notin f(\emptyset)\} \cup \{y|y \in f(\emptyset)\} & \text{Die, } \{\mathbf{0}\}\text{! DIE!} \\
= & \{y|(\exists x \in X \cdot y \in f(\{x\}) \wedge y \notin f(\emptyset)) \vee y \in f(\emptyset)\} & \text{group together} \\
= & \{y|(\exists x \in X \cdot y \in f(\{x\})) \vee y \in f(\emptyset)\} & \text{absorb} \\
= & f(X) & \text{by the above}
\end{array}
$$