1. We start with \((s_p, d_1) \rightarrow (n, d_2)\) = \((s_p, g) \rightarrow (e_p, g)\). Since \(n = e_p\), we match the case on line [21].

Now, \(callers(p) = \{n_2, n_7\}\). Since the order is unspecified on line [22], we'll process \(n_2\) first.

Line [23] has us search for calls to procedure \(p\) from \(n_2\) and returns from \(p\) into \(\text{returnSite}(n_2)\) associated with \(g\). \(\text{returnSite}(n_2) = n_3\). Since \(\langle n_2, g \rangle \rightarrow \langle s_p, g \rangle\) and \(\langle e_p, g \rangle \rightarrow \langle n_3, g \rangle\) are the only ones that match (consulting \(E^#\) in Figure 2), we get \(d_4 = g\) and \(d_5 = g\) on line [23].

By assumption, the check on line [24] passes, and on line [25], we add the new summary edge \(\langle n_2, g \rangle \rightarrow \langle n_3, g \rangle\). Line [26] then asks us for \(d_3\)'s such that \(\langle s_{main}, d_3 \rangle \rightarrow \langle n_7, g \rangle\). Consulting Figure 2 again, the only candidate is \(d_3 = 0\), so we may call \text{Propagate()} on \(\langle s_{main}, d_3 \rangle \rightarrow \langle n_3, g \rangle\). Looking at Figure 2, we see that \(\langle n_3, g \rangle\) is indeed labelled as reachable.

Now consider \(c = n_7\). \(\text{returnSite}(n_7) = n_8\), and we get \(d_4 = d_5 = g\). We thus add \(\langle n_7, g \rangle \rightarrow \langle n_8, g \rangle\) to our set of summary edges.

There is no \(d_3\) such that \(\langle s_p, d_3 \rangle \rightarrow \langle n_7, g \rangle\); hence, we don’t call \text{Propagate()} at all. This is consistent with Figure 2: although there is an edge in \(E^#\) from \(\langle e_p, g \rangle\), which is reachable, to \(\langle n_8, g \rangle\), which is not, the “possibly-uninitializedness” is not propagated along it.

2. The representation function for \(f(X) = D - X\) is

\[\{(0, 0), (0, a), (0, b), (0, c)\}\]

\([R_f] \neq f\). To see this, note that \(f(\{a\}) = \{b, c\}\), but \([R_f](\{a\}) = \{a, b, c\}\).

3. The proof is given in the RHS tech report at \texttt{ftp://ftp.diku.dk/diku/semantics/papers/D-215.ps.Z}. Let \(D\) be given, and assume that \(f\) distributes over \(\cup\). Let \(X \subseteq D\). Then,
\[ f(X) \]
\[ = \bigcup_{x \in X} f(\{x\}) \cup f(\emptyset) \quad \text{since } f \text{ is distributive} \]
\[ = \bigcup_{x \in X} \{y | y \in f(\{x\})\} \cup \{y | y \in f(\emptyset)\} \]
\[ = \bigcup_{x \in X} \{y | y \in f(\{x\}) \lor y \in f(\emptyset)\} \]
\[ = \{y | \exists x \in X \cdot y \in f(\{x\}) \lor y \in f(\emptyset)\} \]

Now, using the above, we write,

\[ [[R_f]](X) \]
\[ = \{y | \exists x \in X \cdot (x, y) \in R_f\} \cup \{y | (0, y) \in R_f\} - \{0\} \quad \text{definition of } [[R_f]] \]
\[ = \{y | \exists x \in X \cdot y \in f(\{x\}) \land y \notin f(\emptyset)\} \cup \{y | (0, y) \in R_f\} - \{0\} \quad \text{part of definition of } R_f \]
\[ = \{y | \exists x \in X \cdot y \in f(\{x\}) \land y \notin f(\emptyset)\} \cup \{y | y \in f(\emptyset) \lor y = 0\} - \{0\} \quad \text{other part of definition of } R_f \]
\[ = \{y | \exists x \in X \cdot y \in f(\{x\}) \land y \notin f(\emptyset)\} \cup \{y | y \in f(\emptyset) \lor (0, y) \in R_f\} - \{0\} \quad \text{re-arrange} \]
\[ = \{y | \exists x \in X \cdot y \in f(\{x\}) \land y \notin f(\emptyset)\} \cup \{y | y \in f(\emptyset)\} \quad \text{Die, } \{0\}! \text{ DIE!} \]
\[ = \{y | (\exists x \in X \cdot y \in f(\{x\}) \land y \notin f(\emptyset)) \lor y \in f(\emptyset)\} \quad \text{group together} \]
\[ = \{y | (\exists x \in X \cdot y \in f(\{x\})) \lor y \in f(\emptyset)\} \quad \text{absorb} \]
\[ = f(X) \quad \text{by the above} \]