CSC 2125, Fall 2006
Homework 1
Bring solutions to class on Sept. 26

1. Let S be a finite set, and let L be the lattice of subsets of S, with order $\subseteq$. Show that any function $f(x)$ constructed from union, intersection, and constant sets is monotonic. Here, I mean that $f(x) = e$ where e can be specified by the grammar

$$e ::= x \mid S' \mid e \cup e \mid e \cap e$$

   where S' is any subset of S. Your proof should be by induction on the structure of e.

2. Suppose that we extend the grammar for e from problem 1 to include the complement operator $!e$, where $!T = S - T$. Is f still guaranteed to be monotonic? If it is, justify your answer. If it's not, explain why a transfer function defined by $Out(stmt) = Gen(stmt) \cup (In(stmt) - Kill(stmt))$, which seems to include negation, is monotonic.

3. Let A be a lattice, with order $\leq$. Define $A \to A$ to be the set of all functions from A to A, and define $f \leq' g$ if $f(x) \leq g(x)$ for all $x \in A$.

   - Show that $A \to A$ with order $\leq'$ is also a lattice. That is, show that for all $f, g \in A \to A$, $f \sqcup g$ and $f \sqcap g$ always exist.
   - Suppose lattice A has height h and that A is finite with n elements. What is the height of the lattice $(A \to A, \leq')$? (When counting height, count "edges" rather than "nodes," e.g., if A were the lattice $\{a, b\}$ with $a < b$, then its height would be 1.)

4. Exercise 2.10 from NNH. (This problem is to show that MFP(6) is different from MOP(6) for a constant propagation question.)

5. Exercise 2.14 from NNH. (This problem is to compare two Monotone Frameworks for Detection of Signs Analysis.)