Model-Checking Frameworks: Outline

- Theory (Part 1)
  - Notion of Abstraction
  - Aside: over- and under-approximation, simulation, bisimulation
  - Counter-example-based abstraction refinement

- Abstraction and abstraction refinement in program analysis (Part 2)
  - Kinds of abstraction:
    - Data, predicate
  - Building abstractions
    - Aside: weakest precondition
  - Counter-example-based abstraction refinement

Outline, cont’d

- 3-valued abstraction and abstraction-refinement (Part 3)
  - 3-valued logic
  - Theory of 3-valued abstractions: combining over- and under-approximation
  - 3-valued model-checking
  - Building 3-valued abstractions
  - Counter-example-based abstraction refinement
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Model Checking

Given a:

- Finite transition system $M(S, s_0, R, L)$
- A temporal property $\varphi$

The model checking problem:

Does $M$ satisfy $\varphi$?

$M \models \varphi$
Model Checking

Temporal properties:

- “Always x=y”
  \( G(x=y) \)
- “Every Send is followed immediately by Ack”
  \( G(Send \rightarrow X\text{ Ack}) \)

\[ \text{“Safety” properties} \]

- “Reset can always be reached”
  \( GF \text{ Reset} \)
- “From some point on, always switch on”
  \( FG \text{ switch on} \)

\[ \text{“Liveness” properties} \]

Model Checking (safety)

Add reachable states until reaching a fixed-point

\( \bigcirc = \text{bad state} \)
Model Checking (safety)

Too many states to handle!

\( \circ = \text{bad state} \)

Abstraction

\[ \alpha : S \rightarrow S' \]
Abstraction Function: A Simple Example

- Partition variables into visible(□) and invisible(□) variables.

- The abstract model consists of □ variables. □ variables are made inputs.

- The abstraction function maps each state to its projection over □.

Abstraction Function: Example

Group concrete states with identical visible part to a single abstract state.
Computing Abstractions

- $S$ – concrete state space
- $S'$ – abstract state space
- $\alpha$: $S \rightarrow S'$ - abstraction function
- $\gamma$: $S' \rightarrow S$ - concretization function
- Properties of $\alpha$ and $\gamma$:
  - $\alpha(\gamma(A)) = A$, for $A$ in $S'$
  - $\gamma(\alpha(S)) \supseteq S$, for $S$ in $S$
- The above properties mean that $\alpha$ and $\gamma$ are Galois-connected

Aside: simulations

$M = (s_0, S, R, L)$
$M' = (t_0, S', R', L')$

Definition: $p$ is a simulation between $M$ and $M'$ if
1. $(s_0, t_0) \in p$
2. $\forall (t, t_1) \in R' \exists (s, s_1) \in R$ s.t. $(s, t) \in p$ and $(s_1, t_1) \in p$

Intuitively, every transition in $M'$ corresponds to some transition in $M$
Aside: bisimulation

\( M = (s_0, S, R, L) \)
\( M' = (t_0, S', R', L') \)

Definition: \( p \) is a bisimulation between \( M \) and \( M' \) if
1. \( p \) is a simulation between \( M \) and \( M' \)
2. \( p \) is a simulation between \( M' \) and \( M \)

Computing Existential Transition Relation

\( R^\exists [\text{Dams'97}]: (t, t_1) \in R' \text{ iff } \exists s \in \gamma(t) \text{ s.t. } \exists s_1 \in \gamma(t_1) \text{ and } (s, s_1) \in R \)

This ensures that \( M' \) is the over-approximation if \( M \), or \( M' \) simulates \( M \).
Existential Abstraction (Over-Approximation)

Model Checking Abstract Model

- Let $\varphi$ be a universally-quantified property (i.e., expressed in LTL or ACTL) and $M'$ simulates $M$

- Preservation Theorem
  $\models M' \models \varphi \rightarrow M \models \varphi$

- Converse does not hold
  $M' \not\models \varphi \not\rightarrow M \not\models \varphi$

- The counterexample may be spurious
Computing Transition Relation

- $R^\exists [\text{Dams'97}]: (t, t_1) \in R' \iff \forall s \in \gamma(t) \exists s_1 \in \gamma(t')$ and $(s, s_1) \in R$
- This ensures that $M'$ is the under-approximation if $M$, or $M$ simulates $M'$.

Universal Abstraction (Under-Approximation)
Model Checking Abstract Model

- Let $\varphi$ be a existential-quantified property (i.e., expressed in ECTL) and $M$ simulates $M'$

- Preservation Theorem
  
  $M' \models \varphi \rightarrow M \models \varphi$

- Converse does not hold
  
  $M' \not\models \varphi \not\rightarrow M \models \varphi$

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Model Checking Abstract Model

$M = (s_0, S, R, L)$ and $M' = (t_0, S', R', L')$ related by bisimulation

Then, for every CTL/LTL property $\varphi$:

$M' \models \varphi \rightarrow M \models \varphi$

$M' \not\models \varphi \rightarrow M \not\models \varphi$

So, why not use bisimulation for abstraction?
Our specific problem

- Let $\varphi$ be a universally-quantified property (i.e., expressed in LTL or ACTL) and $M'$ simulates $M$

- Preservation Theorem
  \[ M' \models \varphi \rightarrow M \models \varphi \]

- Converse does not hold
  \[ M' \not\models \varphi \rightarrow M \not\models \varphi \]

- The counterexample may be spurious

Checking the Counterexample

- Counterexample : $(c_1, ..., c_m)$
  - Each $c_i$ is an assignment to $V$.

- Simulate the counterexample on the concrete model.
Checking the Counterexample

Concrete traces corresponding to the counterexample:

\[ \phi = I(s_1) \land (\text{Initial State } s_0 \text{ in our case}) \]

\[ \bigwedge_{i=1}^{m-1} R(s_i, s_{i+1}) \land (\text{Unrolled Transition Relation}) \]

\[ \bigwedge_{i=1}^{m} \text{visible}(s_i) = c_i \quad (\text{Restriction of } \square \text{ to Counterexample}) \]

Abstraction-Refinement Loop

\[ M, \varphi, \alpha \rightarrow M', \varphi \rightarrow \text{Pass} \rightarrow \text{No Bug} \]

\[ \alpha' \

\[ \text{Refine} \rightarrow \text{Spurious} \rightarrow \text{Check Counterexample} \rightarrow \text{Real} \rightarrow \text{Bug} \]
Refinement methods...

**Localization**
(R. Kurshan, 80’s)

![Diagram showing localization concepts: Frontier, Visible, Invisible, Inputs, and a variable φ.]

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Refinement methods...

**Abstraction/refinement with conflict analysis**
(Chauhan, Clarke, Kukula, Sapra, Veith, Wang, FMCAD 2002)

- Simulate counterexample on concrete model with SAT
- If the instance is unsatisfiable, analyze conflict
- Make visible one of the variables in the clauses that lead to the conflict
Why spurious counterexample?

Refinement

Problem: Deadend and Bad States are in the same abstract state.
Solution: Refine abstraction function.
The sets of Deadend and Bad states should be separated into different abstract states.
Refinement

\[ \alpha' \]

Refinement: \( \alpha' \)

Refinement

\[ \phi_D = I(s_1) \land \bigwedge_{i=1}^{f-1} R(s_i, s_{i+1}) \land \bigwedge_{i=1}^{f} \text{visible}(s_i) = c_i \]
\( \phi_B = R(s_f, s_{f+1}) \land \) visible\((s_f) = c_f \land \) visible\((s_{f+1}) = c_{f+1} \)
Refinement as Separation

\[
\begin{array}{c|cccc}
  & d_1 & b_1 & b_2 \\
 0 & 1 & 0 & 0 & 1 & 0 & 1 \\
 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
 0 & 1 & 0 & 0 & 1 & 1 & 1 \\
\end{array}
\]

Refinement: Find subset \( \Box \) of \( \Box \) that separates between all pairs of deadend and bad states. Make them visible.

Keep \( \Box \) small!

Refinement as Separation

The state separation problem
Input: Sets D, B
Output: Minimal \( \Box \) s.t.:
\[\forall d \in D, \forall b \in B, \exists u \in \Box. \quad d(u) \neq b(u)\]

The refinement \( \alpha' \) is obtained by adding \( \Box \) to \( \Box \).
Two separation methods

- **ILP-based separation**
  - Minimal separating set.
  - Computationally expensive.

- **Decision Tree Learning based separation.**
  - Not optimal.
  - Polynomial.

We will not talk about these in class.