Lecturer : Christina Christara, ccc@cs.toronto.edu
Lectures : Friday 12-2 PM Room BA 1230
Office Hours : Wednesday 1-2 PM, or by appointment, Room Virtual
Web site : http://www.cs.toronto.edu/~ccc/Courses/2321.html

Topics to be covered

- Introduction – Motivation – Vectors and matrices – Eigenvalues and eigenvectors – Norms and inner products – Block matrices – Boundary value problems (1D) and stencils – Boundary value problems (2D) and stencils – Tensor products of matrices
- Direct methods for solving sqaure linear systems – Gauss elimination, LU factorisation, back and forward substitutions – Symmetric matrices, symmetric positive definite matrices, Cholesky factorisation – Banded matrices – Pivoting – Sparse matrix storage schemes – Adjacency graphs and irreducibility
- Iterative methods for solving linear systems – Introduction – Richardson, Jacobi, Gauss-Seidel, SOR and SSOR methods – Block methods – Convergence of matrices and vectors – Convergence of iterative methods – Rate of convergence of iterative methods – Convergence theorems: Comparison of Jacobi and GS, Diagonal dominant matrices, SPD matrices, Spectral radius of SOR, Consistently ordered matrices - Optimal ω for SOR – Rates of convergence of basic iterative methods on the model problem, Computational issues – Preconditioning – Symmetrisable and extrapolated methods – Polynomial acceleration
- Chebyshev acceleration (briefly)
- Non-square linear systems and linear least squares solution – Overdetermined systems, underdetermined systems – Linear least squares solution – Normal equations – QR factorization – Gram-Schmidt orthogonalization
- Conjugate gradient acceleration – The steepest descent method – The family of Conjugate Direction methods – The Conjugate Gradient method – A three-term recurrence relation for CG – The preconditioned CG method
- Partial Differential Equations – Schur complement method, arrowhead matrix, application to the 1D BVP – The use of CG for the solution of the Schur complement system – Schur complement method, arrowhead matrix, application to the 2D BVP – Schwarz alternating (splitting) method, preconditioning – Multigrid method, two- and multi-level method, preconditioning, extension and restriction operators, convergence, V-cycle and full MG – Fast Fourier Transform methods, application to the 1D BVP – FFT methods for the 2D BVP; diagonalization and block-diagonalization
- Interpolation – Deboor decomposition
- Iterative methods for general (including non-symmetric) systems – Introduction - Krylov subspace methods – Generalized Minimal Residual (GMRES) method – Restarted Generalized Minimal Residual method (GMRES(m)) – Convergence of GMRES – Full Orthogonalization Method (FOM) – Conjugate Residual (CR) method – Other methods (GCR, Orthomin, Orthodir) – Bi-orthogonal bases and related methods (BiCG, QMR, CGS, BiCGStab, TFQMR)

Aims of course

- Review the basic concepts in the numerical solution of linear systems.
- Introduce the state-of-the-art developments in numerical linear algebra / PDEs.
- Develop efficient linear solvers.
- Implement the solvers as software.
- Use of existing software (routines and higher level environments).
- Study the performance of methods and software.

Prerequisites

- Your own will to learn.
- Numerical Linear Algebra (e.g. CSC336/350): some knowledge of direct methods for solving linear systems. Fluency in matrix and vector manipulation, both algebraic and algorithmic. Sparse matrices.
- Interpolation (included in CSC436/351): spline interpolation.
- Partial Differential Equations: minimal knowledge on PDEs.

- Theory of Computer Algorithms: minimal knowledge of computer algorithms, data structures and computational complexity.
- Programming: proficiency in some programming language, preferably MATLAB, FORTRAN or C

Tentative marks distribution

Assignment 1	Due Wednesday, February 9	25%
Term test	Friday, March 4	25%
Assignment 2	Due Friday, March 18	25%
Assignment 3	Due Friday, April 8	25%

- The final marks distribution will be announced around mid-February.
- Term test: Calculators are the only aids permitted.
- The assignments include substantial computer work.
- Assignments are to be done **individually** and expected to look like short reports, i.e., the presentation of the subject counts too.

References

Yousef Saad
Iterative Methods for Sparse Linear Systems
SIAM 2003 (PWS 1996)
`http://www-users.cs.umn.edu/~saad/books.html`

L. A. Hageman and D. M. Young
Applied Iterative Methods
Academic Press 1981

Gene H. Golub and Charles Van Loan
Matrix computations
John Hopkins University Press 1996

Richard S. Varga
Matrix iterative analysis
Prentice Hall 1962

David M. Young
Iterative Solution of Large Linear Systems
Academic Press 1971

Wolfgang Hackbusch
Iterative Solution of Large Sparse Systems of Equations
Springer Verlag, 1994

William L. Briggs
A multigrid tutorial
SIAM 2000 (1987)

Charles Van Loan
Computational Frameworks for the Fast Fourier Transform
SIAM 1992

James M. Ortega
Introduction to Parallel and Vector Solution of Linear Systems
Plenum Press 1988

Gene H. Golub and James M. Ortega
Scientific computing: an introduction with parallel computing
Academic Press 1993

James M. Ortega
Matrix theory: a second course
Plenum Press 1987

Gilbert W. Stewart
Introduction to matrix computations
Academic Press 1973

James M. Ortega
Numerical Analysis: A second course
Academic Press 1972

William F. Ames
Numerical Methods for Partial Differential Equations
Academic Press 1992 3rd edition

C. A. Hall and T. A. Porsching
Numerical Analysis of Partial Differential Equations
Prentice Hall 1990

O. Axelsson and V. A. Barker
Finite element solution of boundary value problems
Academic Press 1984

P. M. Prenter
Splines and Variational Methods
John Wiley & Sons 1975

William W. Hager
Applied Numerical Linear Algebra
Prentice Hall 1988

David Kincaid and Ward Cheney
Numerical Analysis
Brooks/Cole 2002 (1996)

Samuel D. Conte and Carl de Boor
Elementary Numerical Analysis
McGraw-Hill Inc.

L. W. Johnson and R. D. Riess
Numerical Analysis
Addison Wesley

Christina Christara and Winky Wai
A brief introduction to MATLAB
December 2001, September 2011
`http://www.cs.toronto.edu/~ccc/`

Selected papers

Academic integrity

Assignments, homeworks and exams must be your own individual work and using only course materials. While students at your level are well aware of what academic integrity means, please note that violating academic integrity includes more things than presenting others' work as one's own. For example, *not taking reasonable measures to protect your work from being plagiarized by others is also a violation of academic integrity*. This is becoming particularly important now, when so many things are online.

You should *never post anywhere or share with anyone* assignments (or parts thereof), exams (or parts thereof) or solutions (or parts thereof), *even after the deadline*.

Additional information

Assignments will be submitted electronically; details to be given with each assessment.

Assignments will be (highly preferably) typed in latex. A template is given in the course website. Other document processors are acceptable, as long as they produce pdf output. If an assignment is *very cleanly* handwritten and scanned *on a proper scanner* as a single pdf file, and *not photographed*, then it is also acceptable. Photographed assignments will receive 0 marks.

The test will be synchronous and in person.

Office hours will be available remotely with pre-arrangement, either at default times posted, or at other mutually agreeable times. Office hours will be for individual students, not for a group of students. If the University permits, I will also hold in person office hours in a pre-arranged room, for a small number of students who request it.