CSC2321F

Lecturer	: Christina Christara, ccc@cs.toronto.edu
Lectures	: Wednesday 2-4 PM Room SU 255

Office Hours : Friday 1-2 PM, or by appointment, Room BA4226 (remote also possible with pre-arrangement)

Web site : http://www.cs.toronto.edu/~ccc/Courses/2321.html

Topics to be covered

- Introduction Motivation Vectors and matrices Eigenvalues and eigenvectors Norms and inner products Block matrices Boundary value problems (1D) and stencils Boundary value problems (2D) and stencils Tensor products of matrices
- Direct methods for solving squure linear systems Gauss elimination, LU factorisation, back and forward substitutions – Symmetric matrices, symmetric positive definite matrices, Cholesky factorisation – Banded matrices – Pivoting – Sparse matrix storage schemes – Adjacency graphs and irreducibility
- Iterative methods for solving linear systems Introduction Richardson, Jacobi, Gauss-Seidel, SOR and SSOR methods – Block methods – Convergence of matrices and vectors – Convergence of iterative methods – Rate of convergence of iterative methods – Convergence theorems: Comparison of Jacobi and GS, Diagonal dominant matrices, SPD matrices, Spectral radius of SOR, Consistently ordered matrices - Optimal ω for SOR – Rates of convergence of basic iterative methods on the model problem, Computational issues – Preconditioning – Symmetrisable and extrapolated methods – Polynomial acceleration
- Chebyshev acceleration (briefly)
- Non-square linear systems and linear least squares solution Overdetermined systems, underdetermined systems Linear least squares solution Normal equations QR factorization Gram-Schmidt orthogonalization
- Conjugate gradient acceleration The steepest descent method The family of Conjugate Direction methods The Conjugate Gradient method A three-term recurrence relation for CG The preconditioned CG method
- Partial Differential Equations Schur complement method, arrowhead matrix, application to the 1D BVP The use of CG for the solution of the Schur complement system – Schur complement method, arrowhead matrix, application to the 2D BVP – Schwarz alternating (splitting) method, preconditioning – Multigrid method, two- and multi-level method, preconditioning, extension and restriction operators, convergence, V-cycle and full MG – Fast Fourier Transform methods, application to the 1D BVP – FFT methods for the 2D BVP; diagonalization and block-diagonalization
- Interpolation Deboor decomposition
- Iterative methods for general (including non-symmetric) systems Introduction Krylov subspace methods Generalized Minimal Residual (GMRES) method – Restarted Generalized Minimal Residual method (GMRES(m)) – Convergence of GMRES – Full Orthogonalization Method (FOM) – Conjugate Residual (CR) method – Other methods (GCR, Orthomin, Orthodir) – Bi-orthogonal bases and related methods (BiCG, QMR, CGS, BiCGStab, TFQMR)

Aims of course

- Review the basic concepts in the numerical solution of linear systems.
- Introduce recent developments in numerical linear algebra / PDEs.
- Develop efficient linear solvers.
- Implement the solvers as software.
- Use of existing software (routines and higher level environments).
- Study the performance of methods and software.

Prerequisites

- Your own will to learn.
- Good sense of computational errors (CSC336).
- Numerical Linear Algebra (e.g. CSC336): some knowledge of direct methods for solving linear systems. Fluency in matrix and vector manipulation, both algebraic and algorithmic. Sparse matrices.
- Interpolation (included in CSC436): spline interpolation.

- Partial Differential Equations: minimal knowledge on PDEs.
- Theory of Computer Algorithms: minimal knowledge of computer algorithms, data structures and computational complexity.
- Programming: proficiency in some programming language, preferably MATLAB, FORTRAN, C or C++

Tentative marks distribution

References

Assignment 1	Due Monday, October 9, 2023, 6 PM	25%
Term test	Wednesday, October 25, 2023, 2-4 PM	25%
Assignment 2	Due Monday, November 13, 2023, 6 PM	25%
Assignment 3	Due Friday, December 8, 2023, 6 PM	25%

- The final marks distribution will be announced around mid-October.
- Term test: Calculators are the only aids permitted.
- The assignments include substantial computer work.
- Assignments are to be done **individually** and expected to look like short reports, i.e., the presentation of the subject counts too.

Yousef SaadGilberIterative Methods for Sparse Linear SystemsIntrodSIAM 2003 (PWS 1996)Acadhttp://www-users.cs.umn.edu/~saad/books.html

L. A. Hageman and D. M. Young Applied Iterative Methods Academic Press 1981

Gene H. Golub and Charles Van Loan Matrix computations John Hopkins University Press 1996

Richard S. Varga Matrix iterative analysis Prentice Hall 1962

David M. Young Iterative Solution of Large Linear Systems Academic Press 1971

Wolfgang Hackbusch Iterative Solution of Large Sparse Systems of Equations Springer Verlag, 1994

William L. Briggs A multigrid tutorial SIAM 2000 (1987)

Charles Van Loan Computational Frameworks for the Fast Fourier Transform SIAM 1992

James M. Ortega Introduction to Parallel and Vector Solution of Linear Systems Plenum Press 1988

Gene H. Golub and James M. Ortega Scientific computing: an introduction with parallel computing Academic Press 1993

James M. Ortega Matrix theory: a second course Plenum Press 1987 Gilbert W. Stewart Introduction to matrix computations Academic Press 1973

James M. Ortega Numerical Analysis: A second course Academic Press 1972

William F. Ames Numerical Methods for Partial Differential Equations Academic Press 1992 3rd edition

C. A. Hall and T. A. Porsching Numerical Analysis of Partial Differential Equations Prentice Hall 1990

O. Axelsson and V. A. Barker Finite element solution of boundary value problems Academic Press 1984

P. M. Prenter Splines and Variational Methods John Wiley & Sons 1975

William W. Hager Applied Numerical Linear Algebra Prentice Hall 1988

David Kincaid and Ward Cheney Numerical Analysis Brooks/Cole 2002 (1996)

Samuel D. Conte and Carl de Boor Elementary Numerical Analysis McGraw-Hill Inc.

L. W. Johnson and R. D. Riess Numerical Analysis Addison Wesley

Christina Christara and Winky Wai A brief introduction to MATLAB December 2001, September 2011 http://www.cs.toronto.edu/~ccc/

Selected papers

Academic integrity

Assignments, homeworks and exams must be your own individual work and using only course materials. While students at your level are well aware of what academic integrity means, please note that violating academic integrity includes more things than presenting others' work as one's own. For example, *not taking reasonable measures to protect your work from being plagiarized by others is also a violation of academic integrity*. This is becoming particularly important now, when so many things are online.

You should *never post anywhere or share with anyone* assignments (or parts thereof), exams (or parts thereof) or solutions (or parts thereof), *even after the deadline*.

Additional information

Assignments will be submitted electronically; details to be given with each assessment.

Assignments will be (highly preferably) typed in latex. A template is given in the course website. Other document processors are acceptable, as long as they produce pdf output. If an assignment is *very cleanly* handwritten and scanned *on a proper scanner* as a single pdf file, and *not photographed*, then it is also acceptable. Photographed assignments will receive 0 marks.

The test will be synchronous and in person.

Office hours will be available at my office, either at default times posted, or at other mutually agreeable times. If there is need for remote office hours, this is possible with advance notice.

Office hours will be for individual students, not for a group of students. Please wear a mask before coming in; only one student at a time.

Presentation of assignments

General

Include your name and student id in the front page and underline last name. Use font size 12 or larger. Use fixed width fonts (e.g. Courier) for code and output. *Never* use dark background, for anything.

Tables and code output

Always align output with an appropriate format statement. (Align to match equivalent order digits.) Use exponential format for very large (e.g. condition numbers) numbers and very small numbers (e.g. errors, residuals) Use integer format for number of iterations, grid sizes, etc. Always use headers for columns in tables.

Plots

When we say plot quantity A versus B, we mean A is in the vertical (y) axis and B in the horizontal (x). Always use captions for plots/figures, and proper diacritical marks and legends when drawing more than one line.

Submission

Do NOT submit zip, rar and similar files on MarkUs. Only submit pdf, image (eps, png, etc), text (incl. code, latex), etc.