Topics to be covered

• Introduction – Motivation – Vectors and matrices – Eigenvalues and eigenvectors – Norms and inner products – Block matrices – Boundary value problems (1D) and stencils – Boundary value problems (2D) and stencils – Tensor products of matrices
• Direct methods for solving square linear systems – Gauss elimination, LU factorisation, back and forward substitutions – Symmetric matrices, symmetric positive definite matrices, Cholesky factorisation – Banded matrices – Pivoting – Sparse matrix storage schemes – Adjacency graphs and irreducibility
• Chebyshev acceleration (briefly)
• Non-square linear systems and linear least squares solution – Overdetermined systems, underdetermined systems – Linear least squares solution – Normal equations – QR factorization – Gram-Schmidt orthogonalization
• Conjugate gradient acceleration – The steepest descent method – The family of Conjugate Direction methods – The Conjugate Gradient method – A three-term recurrence relation for CG – The preconditioned CG method
• Partial Differential Equations – Schur complement method, arrowhead matrix, application to the 1D BVP – The use of CG for the solution of the Schur complement system – Schur complement method, arrowhead matrix, application to the 2D BVP – Schwarz alternating (splitting) method, preconditioning – Multigrid method, two- and multi-level method, preconditioning, extension and restriction operators, convergence, V-cycle and full MG – Fast Fourier Transform methods, application to the 1D BVP – FFT methods for the 2D BVP; diagonalization and block-diagonalization
• Interpolation – Deboor decomposition
• Iterative methods for general (including non-symmetric) systems – Introduction - Krylov subspace methods – Generalized Minimal Residual (GMRES) method – Restarted Generalized Minimal Residual method (GMRES(m)) – Convergence of GMRES – Full Orthogonalization Method (FOM) – Conjugate Residual (CR) method – Other methods (GCR, Orthomin, Orthodir) – Bi-orthogonal bases and related methods (BiCG, QMR, CGS, BiCGStab, TFQMR)

Aims of course

• Review the basic concepts in the numerical solution of linear systems.
• Introduce the state-of-the-art developments in numerical linear algebra / PDEs.
• Develop efficient linear solvers.
• Implement the solvers as software.
• Use of existing software (routines and higher level environments).
• Study the performance of methods and software.

Prerequisites

• Your own will to learn.
• Numerical Linear Algebra (e.g. CSC336/350): some knowledge of direct methods for solving linear systems. Fluency in matrix and vector manipulation, both algebraic and algorithmic. Sparse matrices.
• Interpolation (included in CSC436/351): spline interpolation.
• Partial Differential Equations: minimal knowledge on PDEs.
• Theory of Computer Algorithms: minimal knowledge of computer algorithms, data structures and computational complexity.
• Programming: proficiency in some programming language, preferably MATLAB, FORTRAN or C

**Tentative marks distribution**

<table>
<thead>
<tr>
<th>Assignment</th>
<th>Due Date</th>
<th>Percentage</th>
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</thead>
<tbody>
<tr>
<td>Assignment 1</td>
<td>Tuesday, October 8</td>
<td>25%</td>
</tr>
<tr>
<td>Term test</td>
<td>Tuesday, October 22</td>
<td>25%</td>
</tr>
<tr>
<td>Assignment 2</td>
<td>Due Tuesday, November 12</td>
<td>25%</td>
</tr>
<tr>
<td>Assignment 3</td>
<td>Due Tuesday, December 3</td>
<td>25%</td>
</tr>
</tbody>
</table>

- The final marks distribution will be announced around mid-October.
- Term test: Calculators are the only aids permitted.
- The assignments include substantial computer work.
- Assignments are to be done individually and expected to look like short reports, i.e., the presentation of the subject counts too.

**References**

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Selected papers