Topics to be covered

• Introduction – Motivation – Vectors and matrices – Eigenvalues and eigenvectors – Norms and inner products – Block matrices – Boundary value problems (1D) and stencils – Boundary value problems (2D) and stencils – Tensor products of matrices
• Direct methods for solving square linear systems – Gauss elimination, LU factorisation, back and forward substitutions – Symmetric matrices, symmetric positive definite matrices, Cholesky factorisation – Banded matrices – Pivoting – Sparse matrix storage schemes – Adjacency graphs and irreducibility
• Chebyshev acceleration (briefly)
• Non-square linear systems and linear least squares solution – Overdetermined systems, underdetermined systems – Linear least squares solution – Normal equations – QR factorization – Gram-Schmidt orthogonalization
• Conjugate gradient acceleration – The steepest descent method – The family of Conjugate Direction methods – The Conjugate Gradient method – A three-term recurrence relation for CG – The preconditioned CG method
• Partial Differential Equations – Schur complement method, arrowhead matrix, application to the 1D BVP – The use of CG for the solution of the Schur complement system – Schur complement method, arrowhead matrix, application to the 2D BVP – Schwarz alternating (splitting) method, preconditioning – Multigrid method, two- and multi-level method, preconditioning, extension and restriction operators, convergence, V-cycle and full MG – Fast Fourier Transform methods, application to the 1D BVP – FFT methods for the 2D BVP; diagonalization and block-diagonalization
• Interpolation – Deboor decomposition
• Iterative methods for general (including non-symmetric) systems – Introduction - Krylov subspace methods – Generalized Minimal Residual (GMRES) method – Restarted Generalized Minimal Residual method (GMRES(m)) – Convergence of GMRES – Full Orthogonalization Method (FOM) – Conjugate Residual (CR) method – Other methods (GCR, Orthomin, Orthodir) – Bi-orthogonal bases and related methods (BiCG, QMR, CGS, BiCGStab, TFQMR)

Aims of course

• Review the basic concepts in the numerical solution of linear systems.
• Introduce the state-of-the-art developments in numerical linear algebra / PDEs.
• Develop efficient linear solvers.
• Implement the solvers as software.
• Use of existing software (routines and higher level environments).
• Study the performance of methods and software.

Prerequisites

• Your own will to learn.
• Numerical Linear Algebra (e.g. CSC350/336): some knowledge of direct methods for solving linear systems. Fluency in matrix and vector manipulation, both algebraic and algorithmic. Sparse matrices.
• Interpolation (included in CSC351/436): spline interpolation.
• Partial Differential Equations: minimal knowledge on PDEs.
• Theory of Computer Algorithms: minimal knowledge of computer algorithms, data structures and computational complexity.
• Programming: proficiency in some programming language, preferably MATLAB, FORTRAN or C

**Tentative marks distribution**

<table>
<thead>
<tr>
<th>Assignment</th>
<th>Due Date</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assignment 1</td>
<td>Tuesday, October 11</td>
<td>25%</td>
</tr>
<tr>
<td>Term test</td>
<td>Tuesday, October 25</td>
<td>25%</td>
</tr>
<tr>
<td>Assignment 2</td>
<td>Tuesday, November 8</td>
<td>25%</td>
</tr>
<tr>
<td>Assignment 3</td>
<td>Tuesday, December 6</td>
<td>25%</td>
</tr>
</tbody>
</table>

• The final marks distribution will be announced around mid-October.
• Term test: Calculators are the only aids permitted.
• The assignments include substantial computer work.
• Assignments are to be done **individually** and expected to look like short reports, i.e., the presentation of the subject counts too.

**References**

- Yousef Saad
  Iterative Methods for Sparse Linear Systems
  SIAM 2003 (PWS 1996)

- L. A. Hageman and D. M. Young
  Applied Iterative Methods
  Academic Press 1981

- Gene H. Golub and Charles Van Loan
  Matrix computations
  John Hopkins University Press 1996

- Richard S. Varga
  Matrix iterative analysis
  Prentice Hall 1962

- David M. Young
  Iterative Solution of Large Linear Systems
  Academic Press 1971

- Wolfgang Hackbusch
  Iterative Solution of Large Sparse Systems of Equations
  Springer Verlag, 1994

- William L. Briggs
  A multigrid tutorial
  SIAM 2000 (1987)

- Charles Van Loan
  Computational Frameworks for the Fast Fourier Transform
  SIAM 1992

- James M. Ortega
  Introduction to Parallel and Vector Solution of Linear Systems
  Plenum Press 1988

- Gene H. Golub and James M. Ortega
  Scientific computing: an introduction with parallel computing
  Academic Press 1993

- James M. Ortega
  Matrix theory: a second course
  Plenum Press 1987

- Gilbert W. Stewart
  Introduction to matrix computations
  Academic Press 1973

- James M. Ortega
  Numerical Analysis: A second course
  Academic Press 1972

- William F. Ames
  Numerical Methods for Partial Differential Equations

- C. A. Hall and T. A. Porsching
  Numerical Analysis of Partial Differential Equations
  Prentice Hall 1990

- O. Axelsson and V. A. Barker
  Finite element solution of boundary value problems
  Academic Press 1984

- P. M. Prenter
  Splines and Variational Methods
  John Wiley & Sons 1975

- William W. Hager
  Applied Numerical Linear Algebra
  Prentice Hall 1988

- David Kincaid and Ward Cheney
  Numerical Analysis

- Samuel D. Conte and Carl de Boor
  Elementary Numerical Analysis
  McGraw-Hill Inc.

- L. W. Johnson and R. D. Riess
  Numerical Analysis
  Addison Wesley

- Christina Christara and Winky Wai
  A brief introduction to MATLAB
  December 2001, September 2011
  Selected papers