

# CSC 456-2306 - Fall 2022 - Assignment 1

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## 1 Introduction

Latex is a text formatting software. It takes a source file ending in `.tex`, and produces formatted output in pdf or postscript.

### 1.1 1(a)

This is a sample latex source file, ending in `.tex`. You can change fonts to *italics* or to **bold**, or go back to roman. If you want to make comments visible in the source file, but not in the formatted output, precede them with the percent (%) symbol.

### 1.2 1(b)

It is very useful to give labels to just about anything in Latex, so that it can be referred to later in the file.

In this sample file, we define sections and subsections. You can also define subsubsections. Latex numbers these automatically and in a nested way.

Every time you leave a blank line in this source file, latex changes paragraph in the output. But changing line in the source file without leaving a blank line does not change line or paragraph in the output. Keeping your source file lines short helps readability of your source file, therefore, use return/enter reasonably frequently.

## 2 Lots of mathematics

We first given an example of `equation` with a number (label).

The error formula for a polynomial interpolant  $p_n$  of a function  $f \in C^{n+1}$ , on  $n + 1$  distinct points  $x_0, x_1, \dots, x_n$ , is

$$f(x) - p_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!} \prod_{i=0}^n (x - x_i). \quad (1)$$

### 2.1 Relation alignment and labelling

If you want to align math stuff, you can do it with `eqnarray`.

### 2.1.1 Aligning and labelling

In the following, we align  $p_1(0)$  with  $p_1(1)$ , etc, and give a label to each equation:

$$p_1(0) = 0 \tag{2}$$

$$p_1(1) = 10. \tag{3}$$

### 2.1.2 Aligning and selective labelling

If you do not want an equation or a line in eqnarray to be numbered, then use `nonumber`. For example, only the first line of eqnarray will be numbered, by using

$$p_1(0) = 0 \tag{4}$$

$$p_1(1) = 10.$$

### 2.1.3 Aligning without labelling

If, on the other side, you do not want any line in eqnarray to be numbered, then use `eqnarray*`, as in the following example:

$$p_1(0) = 0$$

$$p_1(1) = 10.$$

### 2.1.4 Equations without numbers

Finally, if you want a simple equation without number, then just use square brackets, as in the following:

$$A \equiv \frac{1}{8} \left( \frac{a}{h_x^2} T_{-2}^{E,M} \otimes T_6^{D,N} + \frac{c}{h_y^2} T_6^{E,M} \otimes T_{-2}^{D,N} + \frac{f}{8} T_6^{E,M} \otimes T_6^{D,N} \right).$$

You can refer to a relation to which you have given a label. For example, relation (1) gives the polynomial interpolation error formula.

You can refer to a section by its label too. For example, the introduction is in Section 1.

## 2.2 Other mathematics

Here are examples of more math, with dots notation, fractions, subscripts, superscripts, etc.

Let  $\Delta_x \equiv \{x_i = i/M, i = 0, \dots, M\}$  and  $\Delta_y \equiv \{y_j = j/N, j = 0, \dots, N\}$  be uniform partitions of  $(0, 1)$  with step-sizes  $h_x = \frac{1}{M}$  and  $h_y = \frac{1}{N}$ , respectively. We denote by  $S_{\Delta_x}$  and  $S_{\Delta_y}$  the quadratic spline spaces with respect to partitions  $\Delta_x$  and  $\Delta_y$ , respectively. The basis functions  $\{\phi_i^x(x)\}_{i=1}^M$  and  $\{\phi_j^y(y)\}_{j=1}^N$  for  $S_{\Delta_x}$  and  $S_{\Delta_y}$ , respectively, are generated through appropriate transformations of the model quadratic spline  $\phi(x)$  defined by  $\{\phi(x) \equiv x^2$  for  $0 \leq x \leq 1$ ;  $\phi(x) \equiv -3 + 6x - 2x^2$  for  $1 \leq x \leq 2$ ;  $\phi(x) \equiv 9 - 6x + x^2$  for  $2 \leq x \leq 3$ ;  $\phi(x) \equiv 0$  elsewhere $\}$ .

Table 1: Observed errors and respective orders of convergence by the two-step QSC for Problem 1, for several gridsizes  $N \times N$ .

$N$	error	order
32	1.8e-07	
64	1.1e-08	3.97
128	7.8e-10	3.87

Mathematical functions such as  $\sin(x)$ ,  $\cos(x)$ ,  $\log(x)$ ,  $\exp(x)$ , etc, should be printed in roman fonts. This is why they are preceded by a backslash. Similarly, precede with backslash the symbols max, min, etc.

Integrals and Sums? The  $L_2$  norm of a function  $f(x)$  in the interval  $[a, b]$  is defined by  $\|f\|_{L_2[a,b]} \equiv (\int_a^b f^2(x)dx)^{1/2}$ .

### 3 Matrices

Here are some examples of matrices:

$$E \equiv \begin{bmatrix} \mathbf{I}^M \\ 0 \end{bmatrix} \text{ and } R \equiv \begin{bmatrix} 1 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 1 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & \cdot & \cdot & 0 \\ & & & & & \cdot & \cdot & \cdot & & \\ 0 & \cdot & \cdot & \cdot & \cdot & 0 & 1 & 0 & 0 & 0 \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & 1 & 0 \end{bmatrix},$$

and

$$\begin{bmatrix} \mathbf{L}_{11} & \mathbf{L}_{12} \\ \mathbf{L}_{21} & \mathbf{L}_{22} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \text{ in } \Omega.$$

Using arrays, we can present the definition of the model quadratic spline  $\phi(x)$  given in Section 2.2 in a different way:

$$\psi(x) \equiv \begin{cases} x^2 & \text{if } 0 \leq x \leq 1, \\ -3 + 6x - 2x^2 & \text{if } 1 \leq x \leq 2, \\ 9 - 6x + x^2 & \text{if } 2 \leq x \leq 3, \\ 0 & \text{elsewhere.} \end{cases}$$

This takes more space, but it is easier to read than the one given in Section 2.2.

### 4 Tables

We first show a simple table, like Table 1, then some more complicated tables.

There are several tricks to save space in tables, by shortening the horizontal space between columns. One is shown below. Notice that Tables 3 and 4 present identical data, but the latter saves space.

Table 2: Errors on the gridpoints, respective orders of convergence, number of iterations for convergence of the  $DL$ -preconditioned GMRES method and time in seconds, corresponding to Problem 3 discretized by the QSC method, for several gridsizes  $N \times N$ . The solution of the preconditioner is obtained by 1D-FFTQSC and 2D-FFTQSC.

$N$	$\zeta = -15$								$\zeta = -50$		time total GE
	on gridpoints		no. of iter.		time				no. of iter.		
	error	order	step 1	step 2	per it.		total		step 1	step 2	
					1D-FFTQSC	2D-FFTQSC	1D-FFTQSC	2D-FFTQSC			
32	3.1e-08		18	13	0.003	0.18	0.003	0.18	24	20	0.11
64	1.9e-09	4.00	18	13	0.013	0.73	0.013	0.73	26	20	0.95
128	1.2e-10	4.01	18	13	0.060	3.16	0.063	3.24	26	20	10.51
256	7.4e-12	4.01	18	13	0.281	13.90	0.297	14.31	26	20	162.28
512	4.6e-13	4.01	18	13	1.315	61.58	1.456	65.79	26	20	

Table 3: Number of iterations for convergence of the  $-\hat{\Delta}_h$ -preconditioned GMRES method with  $(\xi_1, \xi_2)$  as shown, corresponding to Problems 5 and 6 discretized by the QSC method, for several gridsizes  $N \times N$ .

$N$	$(\xi_1, \xi_2)$ step	Problem 5										Problem 6							
		(1,0)		(2,1)		(3,1)		(5,1)		(25,1)		(1,0)		(2,1)		(3,1)		(5,1)	
		1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
32		12	9	15	11	12	9	11	8	11	8	21	16	16	12	17	13	19	14
64		12	9	15	11	12	9	11	8	11	8	21	16	16	12	17	13	19	14
128		12	9	15	11	12	9	11	8	11	9	22	16	16	12	18	13	19	14
256		12	9	15	11	12	9	11	8	11	9	22	16	16	12	18	13	19	14

Table 4: Number of iterations for convergence of the  $-\hat{\Delta}_h$ -preconditioned GMRES method with  $(\xi_1, \xi_2)$  as shown, corresponding to Problems 5 and 6 discretized by the QSC method, for several gridsizes  $N \times N$ .

$N$	$(\xi_1, \xi_2)$ step	Problem 5										Problem 6							
		(1,0)		(2,1)		(3,1)		(5,1)		(25,1)		(1,0)		(2,1)		(3,1)		(5,1)	
		1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
32		12	9	15	11	12	9	11	8	11	8	21	16	16	12	17	13	19	14
64		12	9	15	11	12	9	11	8	11	8	21	16	16	12	17	13	19	14
128		12	9	15	11	12	9	11	8	11	9	22	16	16	12	18	13	19	14
256		12	9	15	11	12	9	11	8	11	9	22	16	16	12	18	13	19	14

Remember that you can refer to tables by their labels. For example, Table 1 is the first (simple) table and Table 4 the last table.

## 5 Figures

### 5.1 Postscript and other image formats

We can easily include figures that are in encapsulated postscript. Files in encapsulated postscript can be easily obtained through MATLAB by the `> print -depsc file.eps` command. Similarly, you can use portable network graphics (png) files. In the following, we show one single centered figure, and two figures side-by-side, together with their captions.

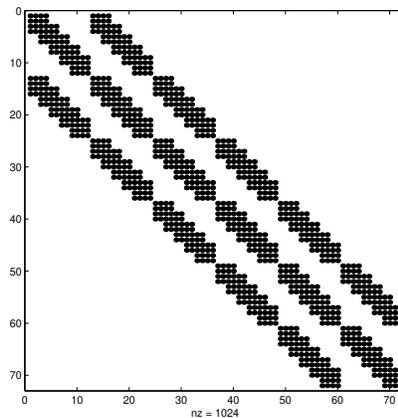


Figure 1: The structure of the QSC matrix arising from the discretization of a system of two PDEs subject to general boundary conditions with the alternating ordering, for  $N = M = 4$ . The number of non-zero entries is denoted by `nz`.

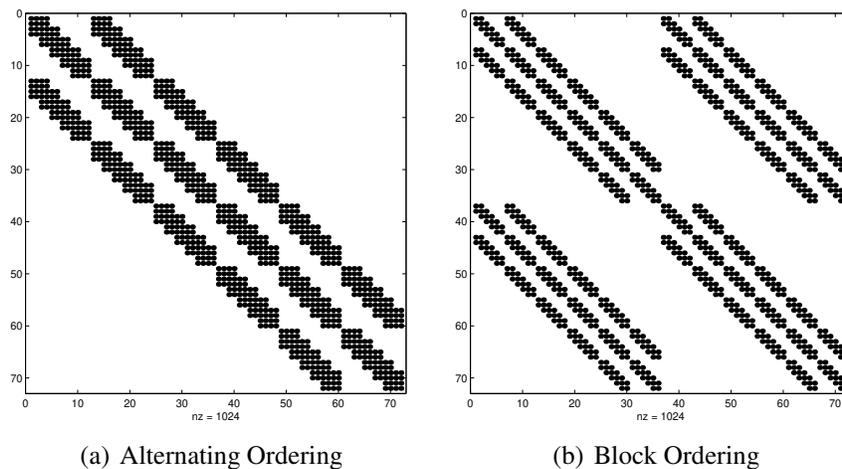


Figure 2: The structure of the QSC matrix arising from the discretization of a system of two PDEs subject to general boundary conditions with the alternating and block orderings, for  $N = M = 4$ . The number of non-zero entries is denoted by `nz`.



## 7 Embedding code and output

We sometimes need to embed code and output into Latex. This should be done using *fixed width fonts*, in order to maintain the original alignment, indentation, etc. There are many ways to include some text with fixed width fonts. One is by using `\tt`, for example, this is fixed width font. Another is by using the verbatim environment with `\begin{verbatim}` and `\end{verbatim}`, such as

```
this is fixed width font
```

With the verbatim environment, everything within the begin and end is taken verbatim (without account for subscripts, other notation, etc).

I find the most convenient way to include code and output already saved in some file is by using the package “listings”, already invoked in the beginning of this sample latex file. Then to include, say file `trochoid1.m`, you can use `\lstinputlisting{trochoid1.m}` as shown and get

```
function [ix, iy] = trochoid1(a, r, U, n)

du = U/n;
x = 0;           % x = a*u0 - r*sin(u0);
y = r;          % y = r*cos(u0);
u = 0;          % u = u0;
ix(1) = round(x);
iy(1) = round(y);
for i = 2:n+1
    u = u + du;
    x = a*u - r*sin(u);
    y = r*cos(u);
    ix(i) = round(x);
    iy(i) = round(y);
end
```

Note that the linespread (vertical space) for the code is less (0.9) than the linespread for the rest of the paper (1.1). This is done by explicitly changing the linespread. While this change in linespread is not necessary, it is useful, as often code takes considerable number of lines, and smaller linespread makes the presentation more concise. Note also that the font size is a little smaller for the included code than the default. This is done by the listings environment automatically. This is also desirable in the case of long code.

## 8 Producing the pdf or postscript file

To get a pdf file from a latex source file, on any Unix/Linux system (i.e. on wolf) you type

```
% pdflatex 456a1.tex
```

This creates a `456a1.pdf` file. Sometimes, you may have to re-run the above a second time, to get rid of some error/warning messages.

To get a postscript file from a latex source file, on any Unix/Linux system (i.e. on wolf) you type

```
% latex 456a1.tex
% latex 456a1.tex
% dvips 456a1
```

This creates a `456a1.ps` file, which can be printed by

```
% lpr 456a1.ps
```

or previewed by

```
% gv 456a1.ps
```

## 9 Preparing the bibliography

You may not need this, but it does not hurt to include an example of using a bibliography. If, in your paper or assignment, you make citations to various papers, books, theses, technical reports, etc, the best way to do this is to create a file, say `assign.bib`, in the format shown in the file provided. Then you can refer to any work by the label corresponding to it as given in the file, for example, the publications [1], [2], [3]. Because of the above citations, an extra (final) section of References appears in the formatted output.

Note that, in the `assign.bib` file, there are certain fields that need to be filled, e.g. author names, title of work, etc, and the author names for a work are separated by *and*. Notice also that only the papers you cite in the Latex file will appear in the references list, independently of how many other works are included in the `assign.bib` file.

However, there is a procedure that helps Latex create pointers to various works you cite, and it must be followed carefully. First, you must create the appropriate `456a1.bbl` file. This is done with the `bibtex` command *after* the first run of the `latex` or `pdflatex` command. Then, the `latex` or `pdflatex` command may have to be run *twice* every time you update the source file.

To summarize, on any Unix/Linux system (i.e. on wolf) you type

```
% latex 456a1.tex
% bibtex 456a1
% latex 456a1.tex
% latex 456a1.tex
```

You need to re-run the `bibtex` command every time you update the file `assign.bib`, or you change (add or remove) some citations in the file `456a1.tex`. You need to run the `latex/pdflatex` command *twice* every time you update the file `456a1.tex`.

The `latex` command creates a `456a1.dvi` file. To get a postscript file which you can preview by `gv` or print by `lpr` you type *after* the `latex` command:

```
% dvips 456a1.dvi
% gv 456a1.ps
% lpr 456a1.ps
```

In general, Latex displays many warnings and other messages. Most of them can be safely ignored. However, if you get warnings about “undefined labels”, “multiply defined labels”, or “undefined references”, you may want to run `latex/pdflatex` (and/or `bibtex`) once more or twice. This should normally fix the problem.

Also remember, that you should run the `lpr` command only if you really want to see a hard copy of the document. Often, previewing with `gv` suffices, and a hard-copy is a waste.

## 10 Links

Official LaTeX webpage: <http://latex-project.org/>

Introduction to LaTeX (not too short):

<http://ctan.mirror.rafal.ca/info/lshort/english/lshort.pdf>

## References

- [1] C. CHRISTARA AND D. M. DANG, *Adaptive and high-order methods for valuing American options*, J. Comput. Finance, 14 (2011), pp. 73–113.
- [2] C. C. CHRISTARA AND N. CHUN-HO LEUNG, *Analysis of quantization error in financial pricing via finite difference methods*, SIAM J. Numer. Anal., 56 (2018), pp. 1731–1757.
- [3] Y. CHEN AND C. CHRISTARA, *Penalty Methods for Bilateral XVA Pricing in European and American Contingent Claims by a PDE Model*, J. Comput. Finance, (2020). to appear.