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UNIVERSITY OF TORONTO
Faculty of Arts and Science

APRIL-MAY EXAMINATIONS 2003

CSC 165H1 S
St. George Campus

Duration — 3 hours

Aids allowed: none

Student Number: |_|_|_|_|_|_|_|_|_|

Last Name: ______________________________

First Name: ______________________________

Lecture Section: ______________ **Instructor:** ________________

*Do **not** turn this page until you have received the signal to start.*
(In the meantime, please fill out the identification section above, and read the instructions below.)

This examination consists of 9 questions on 10 pages (including this one). *When you receive the signal to start, please make sure that your copy of the examination is complete.* If you need more space for one of your solutions, use the reverse side of the page and *indicate clearly the part of your work that should be marked.*

Write your student number at the bottom of pages 2-10 of this test.

1: ______/ 3

2: ______/ 5

3: ______/ 6

4: ______/12

5: ______/14

6: ______/ 6

7: ______/ 4

8: ______/ 5

9: ______/ 7

TOTAL: ______/62

Good Luck!

Question 1. [3 MARKS]

Suppose `a`, `b`, `c` and `d` are boolean variables.
Give a logical expression (not using `if`) that represents the value returned by the following Java code:

```
if (a) {
  return b;
} else if (c) {
  return d;
} else {
  return false;
}
```

Question 2. [5 MARKS]

Part (a) [2 MARKS]

Consider the following property of natural numbers:
$m(x, y, z) =$ "$x * y = z$".
Using the property m, write a sentence that is true if and only if a exactly divides b.

Part (b) [3 MARKS]

Let $P(x)$ denote the property "natural number x is prime."
Write a sentence that says there is a minimum prime number.
Use P, but don't use max or min.

Question 3. [6 MARKS]

Three teens went shopping one afternoon and each bought 2 items.
Al bought a CD and a muffin.
Flo bought a book and a CD.
Lee bought a T-shirt and a postcard.

Let $\mathcal{T} = \{\text{Al, Flo, Lee}\}$ and let $\mathcal{I} = \{\text{book, CD, muffin, postcard, T-shirt}\}$.
For $t \in \mathcal{T}$ and $x \in \mathcal{I}$, let $bought(t, x)$ denote "t bought item x".

Part (a) [1 MARK]

Write the following statement in everyday language: $\exists x \in \mathcal{I}, \forall t \in \mathcal{T}, bought(t, x)$.

Part (b) [1 MARK]

Is the sentence in (a) true or false?

Part (c) [4 MARKS]

Write a single open sentence that is
true for $t =$ Lee, and
false for $t =$ Al and $t =$ Flo, and
does not use any constants (i.e. Al, Flo, Lee, book, CD, muffin, postcard or T-shirt).

Question 4. [12 MARKS]

Let N be the set of positive integers. Consider the sentence:

$$\exists i \in N, \forall j \in N, a_{j+i} = a_j$$

Part (a) [1 MARK]

State whether the sentence is true or false for the following sequence:

$$1, 2, 3, 1, 2, 3, 1, 2, 3, \ldots.$$

Part (b) [11 MARKS]

Formally prove, using the proof structures discussed in class, that the sentence is false for the following sequence:

$$1, 1, 2, 1, 2, 1, 2, 1, 2, \ldots$$

Use the following formula for this sequence:

$$a_n = \begin{cases} 1, & n = 1 \lor n \text{ is even} \\ 2, & n \neq 1 \land n \text{ is odd} \end{cases}.$$

(More room for your solution to question 4(b))

Question 5. [14 MARKS]

Let $\mathcal{F}$ denote the set of all functions with domain N and codomain $R^{\geq 0}$.

For any function $f \in \mathcal{F}$, let

$$\Gamma(f) = \{g \in \mathcal{F} \mid \exists a \in R, \exists d \in R, \forall n \in N, g(n) \leq a \cdot f(n) + d\}.$$

Formally prove, using the proof structures discussed in class, that $\forall f \in \mathcal{F}, O(f) \subseteq \Gamma(f)$.

You may use the following result (recall that R^+ represents the set of positive real numbers):

$$O(f) = \{g \in \mathcal{F} \mid \exists c \in R^+, \exists b \in N, \forall n \in N, (n \geq b \Rightarrow g(n) \leq c \cdot f(n))\}.$$

(More room for you solution to question 5).

Question 6. [6 MARKS]

Consider the following algorithm that evaluates a degree n polynomial at a point x.

```
EVAL(A,n,x)
v = A[0]
y = 1
i = 1
while i<=n do
     y = y*x
     if A[i] != 0 then
       v = v + y * A[i]
     i = i+1
return v
```

Part (a) [2 MARKS]

State a loop invariant for `EVAL` involving A, v and i. Do NOT prove that it is a loop invariant.

Part (b) [4 MARKS]

Let t_E denote the worst case time complexity of `EVAL(A,n,x)` as a function of n, where 1 step is an addition, subtraction, or multiplication. Determine, with justification, t_E. Do NOT use O, Ω or Θ.

Question 7. [4 MARKS]

Consider the floating-point number system with $\beta = 2, p = 3, e_{min} = -3$ and $e_{max} = 3$.
Express the base 10 number 0.6 in this system, using rounding to nearest. Explain your reasoning.
You might find the following useful: $1/8 = 0.125, 1/16 = 0.0625$

Question 8. [5 MARKS]

Let $f : R \rightarrow R$ be defined by $f(x) = 1 - x$.

Part (a) [2 MARKS]

What is the condition number of f?

Part (b) [3 MARKS]

How is your answer to part (a) related to catastrophic cancellation?

Question 9. [7 MARKS]

Formally prove, using the proof structures discussed in class, that for all positive real numbers c and all real numbers x, if $0 < rel(1/c, x) < 1$ then $rel(1/c, 2x - cx^2) < rel(1/c, x)$.
(Recall that $rel(x, x')$ is the relative error when approximating x by x').

Total Marks = 62