

CSC 165H L0201 Midterm 2003
Duration — 50 minutes
Aids allowed: none

Student Number:

Last Name: First Name:

*Do **not** turn this page until you have received the signal to start.*
(Please fill out the identification section above,
and read the instructions below.) *Good Luck!*

This midterm consists of 5 questions on 4 pages (including this one). *When you receive the signal to start, please make sure that your copy is complete.* Write your student number at the bottom of pages 2-4 of this test. If you use any space for rough work, please indicate clearly what you want marked.

1: _____/ 3
2: _____/ 5
3: _____/ 3
4: _____/ 6
5: _____/ 8

TOTAL: _____/25

Question 1. [3 MARKS]

Whether or not the Riemann Hypothesis is true is currently unknown. Suppose a researcher proves that:

(*) The Riemann Hypothesis implies S .

Part (a) [2 MARKS] What does (*) tell us if it is also proven that:

1. S is true.
2. S is false.

Part (b) [1 MARK] What could we find out about the Riemann Hypothesis that would make (*) useful?

Question 2. [5 MARKS]

Consider the following, where p , q and r are sentences.

(S) p and q are equivalent only if r .

Part (a) [1 MARK] Express (S) as a sentence in our precise language.

Part (b) [2 MARKS] Express (S) as a sentence in our precise language without using \Rightarrow or \Leftrightarrow (you do not have to make it simple).

Part (c) [2 MARKS] Draw a Venn diagram with sets for p , q and r . Make sure the sets overlap to divide the diagram into eight regions. Shade in the regions corresponding to where your sentence from (a) is true, and put an "X" in the regions where it is not true.

Question 3. [3 MARKS]

Express the sentence

Program F is faulty if some input causes it to crash.

in our precise language, using the domains:

P = the set programs.

I = the set of inputs.

Define any properties you need.

Question 4. [6 MARKS]

Consider the results of a contest.

Let C = the set of contestants.

Let $beat(c1, c2)$ = “contestant $c1$ scored more than contestant $c2$ ”.

Assume every contestant got a different score.

Part (a) [3 MARKS] Using C and $beat$, give a sentence in our precise language that is equivalent to:

“Contestant c won the contest”

Part (b) [3 MARKS] Using C and $beat$, give a sentence in our precise language that is equivalent to:

“Contestant c came in second-place”

Question 5. [8 MARKS]

Consider the following sentence about a sequence of natural numbers a_1, a_2, a_3, \dots :

$$(*) \exists i \in \mathbb{N}, \forall j \in \mathbb{N}, a_j = a_i \implies a_{j+1} \neq a_i$$

Part (a) [4 MARKS] Give the outline of a proof structure for the sentence.

Part (b) [4 MARKS] For each of the following sequences, state whether the sentence (*) is true or false:

1. 1, 2, 2, 1, 2, 2, 1, 2, 2, 1, 2, 2, 1, ...
2. 1, 1, 2, 1, 2, 3, 1, 2, 3, 4, 1, 2, 3, 4, 5, 1, ...
3. 1, 2, 1, 1, 2, 2, 1, 1, 1, 2, 2, 2, 1, 1, 1, 1, 2, 2, 2, 2, 1, ...
4. 1, 1, 2, 1, 2, 2, 1, 2, 2, 2, 1, 2, 2, 2, 2, 1, ...

Total Marks = 25