CSC 165H L0101 Midterm 2003
Duration — 50 minutes
Aids allowed: none

Student Number: |_|_|_|_|_|_|_|_|_|_|_|

Last Name: _____ **First Name:** _____

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*Do **not** turn this page until you have received the signal to start.*
(Please fill out the identification section above,
and read the instructions below.) *Good Luck!*

This midterm consists of 5 questions on 4 pages (including this one). *When you receive the signal to start, please make sure that your copy is complete.*
Write your student number at the bottom of pages 2-4 of this test.
If you use any space for rough work, please indicate clearly what you want marked.

1: _____/ 3

2: _____/ 5

3: _____/ 3

4: _____/ 6

5: _____/ 8

TOTAL: _____/25

Question 1. [3 MARKS]

Whether or not the statement "P = NP" is true is currently unknown. Suppose a researcher proves that:

(*) If P = NP then S.

Part (a) [2 MARKS] What does (*) tell us if it is also proven that:

1. P = NP

2. P $\neq$ NP

Part (b) [1 MARK] What could we find out about S that would make (*) useful?

Question 2. [5 MARKS]

Consider the following, where p, q and r are sentences.

(S) If p then q, otherwise r.

Part (a) [3 MARKS] Express (S) as a sentence in our precise language.

Part (b) [2 MARKS] Draw a Venn diagram with sets for p, q and r. Make sure the sets overlap to divide the diagram into eight regions. Shade in the regions corresponding to where your sentence from (a) is true, and put an "X" in the regions where it is not true.

Question 3. [3 MARKS]

Express the sentence

No process that wants the resource can proceed.

in our precise language, using the domain:

P = the set of all processes.

Define any properties you need.

Question 4. [6 MARKS]

Consider a company with a president, vice-president, and various other employees, that has a chain of command:

President
Vice-president
Senior project manager
...

Each employee obeys themself and everyone higher up in the chain (but no one else). For example: the senior project manager obeys herself, the vice-president, the president, and no one else.

Let E = the set of employees in this company.
Let $obeys(e1, e2)$ = "employee $e1$ obeys employee $e2$".

Part (a) [3 MARKS] Using E and *obeys* (but not the constants "President", "Vice-president" or "Senior Project Manager"), give a sentence in our precise language that is equivalent to:

Employee e is the president.

Part (b) [3 MARKS] Using E and *obeys* (but not the constants "President", "Vice-president" or "Senior Project Manager"), give a sentence in our precise language that is equivalent to:

Employee e is the vice-president.

Question 5. [8 MARKS]

Consider the following sentence about a sequence of natural numbers $a_1, a_2, a_3, \ldots$:

$$(*)\ \forall i \in N, \exists j \in N, a_i = a_j \wedge a_j = a_{j+1}$$

Part (a) [4 MARKS] Give the outline of a proof structure for the sentence.

Part (b) [4 MARKS] For each of the following sequences, state whether the sentence (*) is true or false:

1. $1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \ldots$

2. $4, 4, 3, 3, 2, 2, 1, 1, 4, 3, 2, 1, 1, 1, 1, 1, \ldots$ (all the rest are ones)

3. $1, 2, 1, 1, 2, 2, 1, 1, 1, 2, 2, 2, 1, 1, 1, 1, 2, 2, 2, 2, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 1, \ldots$

4. $1, 1, 1, 2, 3, 3, 2, 1, 1, 1, 2, 3, 3, 2, 1, 1, 1, 2, 3, 3, 2, \ldots$

Total Marks = 25