

Duration: **50 minutes**
Aids Allowed: **NONE** (in particular, no calculator)

Student Number: _____

Last (Family) Name: _____

First (Given) Name(s): _____

Tutorial Section:	LM-123	LM-155	LM-157	UC-85	UC-87
(circle one)	Annie	Patricio	Jennifer	Nathanael	Babak
	Yuk	Simari	Listgarten	Hyafil	Farzad

Do not turn this page until you have received the signal to start.
(In the meantime, please fill out the identification section above,
and read the instructions below *carefully*.)

This term test consists of 3 questions on 7 pages (including this one), printed on both sides of the paper. *When you receive the signal to start, please make sure that your copy of the test is complete.*

Answer each question directly on the test paper, in the space provided. Some of the pages are “blank” (*i.e.*, they do not contain any question): use them for rough work. If you need more space for one of your solutions, use one of the blank pages and *indicate clearly the part of your work that should be marked.*

If you are unable to answer a question (or part of a question), you will get 20% of the marks for the question (or part of the question) if you state clearly that you do not know how to answer. Note that you will *not* get those marks if your answer contains contradictory statements (such as “I do not know how to answer” followed or preceded by parts of a solution that have not been crossed off).

General Hint: We were careful to leave ample space on the test paper to answer each question.

MARKING GUIDE

1: _____/12

2: _____/28

3: _____/10

BONUS

MARKS: _____/ 1

TOTAL: _____/50

Good Luck!

Question 1. [12 MARKS]

Consider the following sentences about sequences of natural numbers a_0, a_1, a_2, \dots :

$$(S1) \quad \forall i \in \mathbb{N} \exists j \in \mathbb{N}, j < i \wedge a_j = a_i$$

$$(S2) \quad \forall i \in \mathbb{N} \exists j \in \mathbb{N}, j > i \wedge a_j = a_i$$

$$(S3) \quad \forall i \in \mathbb{N} \exists j \in \mathbb{N}, j \neq i \wedge a_j = a_i$$

Part (a) [3 MARKS]

Rewrite each of the sentences above in clear, natural English.

(S1):

(S2):

(S3):

Part (b) [3 MARKS]

Determine whether each sentence above is true or false for the following sequence and circle the appropriate answers:

12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0, 0, 0, ...

(S1) is: TRUE / FALSE / IMPOSSIBLE TO DETERMINE* / I DON'T KNOW (FOR 20%)

(S2) is: TRUE / FALSE / IMPOSSIBLE TO DETERMINE* / I DON'T KNOW (FOR 20%)

(S3) is: TRUE / FALSE / IMPOSSIBLE TO DETERMINE* / I DON'T KNOW (FOR 20%)

Part (c) [3 MARKS]

Determine whether each sentence above is true or false for the following sequence and circle the appropriate answers:

0, 0, 1, 0, 1, 2, 0, 1, 2, 3, 0, 1, 2, 3, 4, 0, 1, 2, 3, 4, 5, ...

(S1) is: TRUE / FALSE / IMPOSSIBLE TO DETERMINE* / I DON'T KNOW (FOR 20%)

(S2) is: TRUE / FALSE / IMPOSSIBLE TO DETERMINE* / I DON'T KNOW (FOR 20%)

(S3) is: TRUE / FALSE / IMPOSSIBLE TO DETERMINE* / I DON'T KNOW (FOR 20%)

Part (d) [3 MARKS]

Determine whether each sentence above is true or false for the following sequence and circle the appropriate answers:

0, 1, 0, 2, 1, 3, 2, 4, 3, 5, 4, 6, 5, 7, 6, 8, 7, 9, 8, 10, 9, ...

(S1) is: TRUE / FALSE / IMPOSSIBLE TO DETERMINE* / I DON'T KNOW (FOR 20%)

(S2) is: TRUE / FALSE / IMPOSSIBLE TO DETERMINE* / I DON'T KNOW (FOR 20%)

(S3) is: TRUE / FALSE / IMPOSSIBLE TO DETERMINE* / I DON'T KNOW (FOR 20%)

*i.e., there is not enough information to determine whether the sentence is true or false

*[This is a “blank” page. Use the space below for rough work.
This page will **not** be marked, unless you clearly indicate the part of your work that should be marked.]*

[This test continues on the next page.]

Question 2. [28 MARKS]

Let P represent the set of all programs.

Let T represent the set of all test cases.

Let $f(p, t)$ represent the sentence: “program p failed test t ”.

Rewrite each of the following sentences symbolically, using only the domains and predicate defined above (in other words, you are *not* allowed to define new predicates in your answers). Of course, you can (and should) use quantifiers and connectives where they are appropriate.

Part (a) [3 MARKS]

“Program r passed test w .”

Part (b) [3 MARKS]

“Program q failed some test.”

Part (c) [3 MARKS]

“No program failed every test.”

Part (d) [3 MARKS]

“Every program that passed test u failed test w .”

Part (e) [3 MARKS]

“Programs r and s passed exactly the same tests.”

Part (f) [3 MARKS]

“Program r failed every test that was failed by some other program.”

Question 2. (CONTINUED)**Part (g)** [10 MARKS]

Suppose that the sentences from parts (a) to (f) on the previous page are all true. For each sentence below, determine its value and circle the appropriate answer. Briefly justify each of your answers.

(Note that it is possible to answer this part even if you did not complete parts (a) to (f).)

“ r passed test w ” is:

TRUE / FALSE / IMPOSSIBLE TO DETERMINE* / I DON'T KNOW (FOR 20%)

Justification:

“ r passed test u ” is:

TRUE / FALSE / IMPOSSIBLE TO DETERMINE* / I DON'T KNOW (FOR 20%)

Justification:

“ s failed test w ” is:

TRUE / FALSE / IMPOSSIBLE TO DETERMINE* / I DON'T KNOW (FOR 20%)

Justification:

“all programs passed test w ” is:

TRUE / FALSE / IMPOSSIBLE TO DETERMINE* / I DON'T KNOW (FOR 20%)

Justification:

“some program failed test w ” is:

TRUE / FALSE / IMPOSSIBLE TO DETERMINE* / I DON'T KNOW (FOR 20%)

Justification:

**i.e.*, there is not enough information to determine whether the sentence is true or false

*[This is a “blank” page. Use the space below for rough work.
This page will **not** be marked, unless you clearly indicate the part of your work that should be marked.]*

[This test continues on the next page.]

Question 3. [10 MARKS]

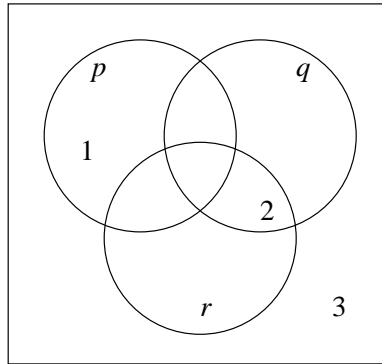
Consider the following sentence:

$$(S) \quad p \text{ and } q \text{ are both true only if } r \text{ is true.}$$

Part (a) [2 MARKS]

Rewrite (S) using symbolic notation.

Next, consider the following diagram illustrating the possible combinations of values for p, q, r :



Part (b) [1 MARK]

Is (S) true or false for the region numbered 1 in the diagram above (circle the appropriate answer)?

TRUE / FALSE / IMPOSSIBLE TO DETERMINE* / I DON'T KNOW (FOR 20%)

Part (c) [1 MARK]

Is (S) true or false for the region numbered 2 in the diagram above (circle the appropriate answer)?

TRUE / FALSE / IMPOSSIBLE TO DETERMINE* / I DON'T KNOW (FOR 20%)

Part (d) [1 MARK]

Is (S) true or false for the region numbered 3 in the diagram above (circle the appropriate answer)?

TRUE / FALSE / IMPOSSIBLE TO DETERMINE* / I DON'T KNOW (FOR 20%)

Part (e) [5 MARKS]

In the diagram above, put an "X" in every region where sentence (S) is false.

*i.e., there is not enough information to determine whether the sentence is true or false

Bonus. [1 MARK]

Write your student number where indicated at the bottom of every odd page, except for page 1. Also, if you have not done so already, complete the identification section at the top of page 1.

Total Marks = 50