Duration: **20 minutes** Aids Allowed: **NONE**

Student Number: _____

Last (Family) Name: _____

First (Given) Name(s): _____

Tutorial Room: _____ **TA's Name:** _____

Prove from the definition, that $n^2 \notin \mathcal{O}(\frac{n^2}{n+1})$.

Negate the definition of $\mathcal{O}$:
$\forall c \in \mathbb{R}^+, \forall B \in \mathbb{N}, \exists n \in \mathbb{N}, n \geq B \wedge f(n) > cg(n)$

Let $c \in \mathbb{R}^+$
Let $B \in \mathbb{N}$
Let $n = B + \lceil c \rceil + 1$
Then $n \geq B$ (By $n = B + \lceil c \rceil + 1$ and $B \geq 0, c > 0$)
$n + 1 > c$ (By $n = B + \lceil c \rceil + 1$)
$n(n+1) > cn$
$n^2(n+1) > cn^2$
$n^2 > c(\frac{n^2}{n+1})$ (Since $n + 1 > 0$)
Since n is a natural number, $\exists n \in \mathbb{N}, n \geq B \wedge n^2 > c(\frac{n^2}{n+1})$
Since B is an arbitrary natural number, $\forall B \in \mathbb{N}, n \geq B \wedge n^2 > c(\frac{n^2}{n+1})$
Since c is an arbitrary positive real number, $\forall c \in \mathbb{R}^+, \forall B \in \mathbb{N}, \exists n \in \mathbb{N}, n \geq B \wedge n^2 > c(\frac{n^2}{n+1})$

Therefore, $n^2 \notin \mathcal{O}(\frac{n^2}{n+1})$