

Duration: 20 minutes

Aids Allowed: NONE

Student Number: _____

Last (Family) Name: _____

First (Given) Name(s): _____

Tutorial Room: _____ TA's Name: _____

Prove from the definition, that $n^2 + n + 2 \in \mathcal{O}(n^2 + 4)$.Let $c = 2$ Then $c \in \mathbb{R}^+$ Let $B = 1$ Then $B \in \mathbb{N}$ Let $n \in \mathbb{N}$ Suppose $n \geq B$, so $n \geq 1$

$$n^2 \leq n^2$$

$$n^2 + 2 \leq n^2 + 4$$

$$n \leq n^2 + 4 \text{ (since } n \geq 1\text{), so } n^2 + n + 2 \leq 2(n^2 + 4)$$

$$n^2 + n + 2 \leq c(n^2 + 4)$$

$$\text{Hence } n \geq B \rightarrow n^2 + n + 2 \leq c(n^2 + 4)$$

$$\text{Since } n \text{ is an arbitrary natural number, } \forall n \in \mathbb{N}, n \geq B \rightarrow n^2 + n + 2 \leq c(n^2 + 4)$$

$$\text{Since } B \text{ is a natural number, } \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \rightarrow n^2 + n + 2 \leq c(n^2 + 4)$$

$$\text{Since } \exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \rightarrow n^2 + n + 2 \leq c(n^2 + 4)$$

Therefore, $n^2 + n + 2 \in \mathcal{O}(n^2 + 4)$