Duration: **20 minutes** Aids Allowed: **NONE**

Student Number: ⌊_|_|_|_|_|_|_|_|_⌋

Last (Family) Name: ______________________________

First (Given) Name(s): ______________________________

Tutorial Room: ____________ **TA's Name:** ____________________

Prove from the definition, that $n^3 + n^2 + 1 \notin \mathcal{O}(n^2)$.

Negate the definition of $\mathcal{O}$:
$\forall c \in \mathbb{R}^+, \forall B \in \mathbb{N}, \exists n \in \mathbb{N}, n \geq B \wedge f(n) > cg(n)$

Let $c \in \mathbb{R}^+$
Let $B \in \mathbb{N}$
Let $n = B + \lceil c \rceil + 1$
Then $n \geq B$
$n > c$ (By $n = B + \lceil c \rceil + 1, B \geq 0, c > 0$)
$n^2 > cn$
$n^3 > cn^2$
By defn of n, $n^2 + 1 \geq 0$, so $n^3 + n^2 + 1 \geq cn^2$
Since n is a natural number, $\exists n \in \mathbb{N}, \geq B \wedge n^3 + n^2 + 1 \geq cn^2$
Since B is an arbitrary natural number, $\forall B \in \mathbb{N}, \exists n \in \mathbb{N}, n \geq B \wedge n^3 + n^2 + 1 \geq cn^2$
Since c is an arbitrary positive real number, $\forall c \in \mathbb{R}^+, \forall B \in \mathbb{N}, \exists n \in \mathbb{N}, n \geq B \wedge n^3 + n^2 + 1 \geq cn^2$

Therefore, $n^3 + n^2 + 1 \notin \mathcal{O}(n^2)$