

Duration: 20 minutes

Aids Allowed: NONE

Student Number: \_\_\_\_\_

Last (Family) Name: \_\_\_\_\_

First (Given) Name(s): \_\_\_\_\_

Tutorial Room: \_\_\_\_\_ TA's Name: \_\_\_\_\_

Prove from the definition, that  $n(n + 1) + 4 \in \mathcal{O}(n^2)$ .Let  $c = 2$ Then  $c \in \mathbb{R}^+$ Let  $B = 3$ Then  $B \in \mathbb{N}$ Let  $n \in \mathbb{N}$ Suppose  $n \geq B$ , so  $n \geq 3$ 

$$n^2 \leq n^2$$

$$n^2 + n^2 \leq 2n^2$$

$$n^2 + n + 4 \leq 2n^2 \text{ (Since } n^2 \geq n + 4 \text{ when } n \geq 3)$$

$$n(n + 1) + 4 \leq cn^2$$

$$\text{Hence } n \geq B \rightarrow n(n + 1) + 4 \leq cn^2$$

Since  $n$  is an arbitrary natural number,  $\forall n \in \mathbb{N}, n \geq B \rightarrow n(n + 1) + 4 \leq cn^2$ Since  $B \in \mathbb{N}, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \rightarrow n(n + 1) + 4 \leq cn^2$ Since  $\exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \rightarrow n(n + 1) + 4 \leq cn^2$