

Duration: 20 minutes

Aids Allowed: NONE

Student Number: _____

Last (Family) Name: _____

First (Given) Name(s): _____

Tutorial Room: _____ TA's Name: _____

Consider the following statement about sequences of natural numbers a_0, a_1, a_2, \dots :

$$(S) \quad \forall i \in \mathbb{N}, \exists j \in \mathbb{N}, \forall k \in \mathbb{N}, k > j \rightarrow a_k \geq a_i$$

Consider the sequence :

$$(A) \quad 0, 0, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, \dots \quad \text{that is defined as: } \forall n \in \mathbb{N}, a_n = \lfloor n/2 \rfloor$$

Prove or disprove (S) for (A).

Solution:

(S) is true for (A).

Let $i \in \mathbb{N}$ Let $j = i + 2$, then $j \in \mathbb{N}$ Let $k \in \mathbb{N}$ Suppose $k > j$, so $k > i + 2$

$$a_k = \lfloor \frac{k}{2} \rfloor$$

$$\geq \lfloor \frac{(i+2)}{2} \rfloor \quad (\text{By } k > i + 2)$$

$$= \lfloor \frac{i}{2} + 1 \rfloor$$

$$= \lfloor \frac{i}{2} \rfloor + 1$$

$$= \lfloor \frac{i}{2} \rfloor$$

$$= a_i \quad (\text{By the defn of A})$$

Then $a_k \geq a_i$ Hence $k > j \rightarrow a_k \geq a_i$ Since k is an arbitrary natural number, $\forall k \in \mathbb{N}, k > j \rightarrow a_k \geq a_i$ Since j is a natural number, $\exists j \in \mathbb{N}, \forall k \in \mathbb{N}, k > j \rightarrow a_k \geq a_i$ Since i is an arbitrary natural number, $\forall i \in \mathbb{N}, \exists j \in \mathbb{N}, \forall k \in \mathbb{N}, k > j \rightarrow a_k \geq a_i$