

Duration: 20 minutes

Aids Allowed: NONE

Student Number: \_\_\_\_\_

Last (Family) Name: \_\_\_\_\_

First (Given) Name(s): \_\_\_\_\_

Tutorial Room: \_\_\_\_\_ TA's Name: \_\_\_\_\_

Consider the following statement about sequences of natural numbers  $a_0, a_1, a_2, \dots$ :

$$(S) \quad \forall i \in \mathbb{N}, \exists j \in \mathbb{N}, \forall k \in \mathbb{N}, k < j \rightarrow a_k \leq a_i$$

Consider the sequence :

$$(A) \quad 0, 0, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, \dots \quad \text{that is defined as: } \forall n \in \mathbb{N}, a_n = \lfloor n/2 \rfloor$$

Prove or disprove (S) for (A).

Solution:

(S) is true for (A).

Let  $i \in \mathbb{N}$ Let  $j = i + 1$ , then  $j \in \mathbb{N}$ Let  $k \in \mathbb{N}$ Suppose  $k > j$ , so  $k > i + 1$ So  $k \geq i$  (By  $k > i + 1$  and  $k \in \mathbb{N}, i \in \mathbb{N}$ )Then  $a_k \geq a_i$  (By defn of A and  $k \geq i$ )Hence  $k < j \rightarrow a_k \leq a_i$ Since  $k$  is an arbitrary natural number,  $\forall k \in \mathbb{N}, k < j \rightarrow a_k \leq a_i$ Since  $j$  is a natural number,  $\exists j \in \mathbb{N}, \forall k \in \mathbb{N}, k < j \rightarrow a_k \leq a_i$ Since  $i$  is an arbitrary natural number,  $\forall i \in \mathbb{N}, \exists j \in \mathbb{N}, \forall k \in \mathbb{N}, k < j \rightarrow a_k \leq a_i$