Duration: **20 minutes** Aids Allowed: **NONE**

Student Number: ⌊_|_|_|_|_|_|_|_|_⌋

Last (Family) Name: ____________________

First (Given) Name(s): ____________________

Tutorial Room: __________ **TA's Name:** ____________________

Consider the following statement about sequences of natural numbers a_0, a_1, a_2, $\ldots$:

$$(S) \quad \forall i \in \mathbb{N}, \exists j \in \mathbb{N}, \forall k \in \mathbb{N}, k < j \rightarrow a_k < a_i$$

Consider the sequence :

$$(A) \quad 0, 0, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, \ldots \quad \text{that is defined as: } \forall n \in \mathbb{N}, a_n = \lfloor n/2 \rfloor$$

Prove or disprove (S) for (A).

Solution:

Let $i \in \mathbb{N}$
  Let $j = 2\lfloor \frac{i}{2} \rfloor$, then $j \in \mathbb{N}$
    Let $k \in \mathbb{N}$
      Suppose $k < j$, so $k < 2\lfloor \frac{i}{2} \rfloor$

        So $\frac{k}{2} < \lfloor \frac{i}{2} \rfloor$ (By $k < 2\lfloor \frac{i}{2} \rfloor$)

        $a_k = \lfloor \frac{k}{2} \rfloor \leq \frac{k}{2} < \lfloor \frac{i}{2} \rfloor = a_i$ (By defn of floor, defn of A)

        Then $a_k < a_i$
      Hence $k < j \rightarrow a_k < a_i$
    Since k is an arbitrary natural number, $\forall k \in \mathbb{N}, k < j \rightarrow a_k < a_i$
  Since j is a natural number, $\exists j \in \mathbb{N}, \forall k \in \mathbb{N}, k < j \rightarrow a_k < a_i$
Since i is an arbitrary natural number, $\forall i \in \mathbb{N}, \exists j \in \mathbb{N}, \forall k \in \mathbb{N}, k < j \rightarrow a_k < a_i$