Duration: **20 minutes** Aids Allowed: **NONE**

Student Number: ___________

Last (Family) Name: ___________

First (Given) Name(s): ___________

Tutorial Room: ___________ **TA's Name:** ___________

Consider the following statement about sequences of natural numbers a_0, a_1, a_2, ...:

$$(S) \quad \forall i \in \mathbb{N}, \exists j \in \mathbb{N}, \forall k \in \mathbb{N}, k > j \rightarrow a_k > a_i$$

Consider the sequence :

$(A) \quad 0, 0, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, \ldots$ that is defined as: $\forall n \in \mathbb{N}, a_n = \lfloor n/2 \rfloor$

Prove or disprove (S) for (A).
Solution:
(S) is true for (A).

Let $i \in \mathbb{N}$
Let $j = i + 2$, then $j \in \mathbb{N}$
Let $k \in \mathbb{N}$
Suppose $k > j$, so $k > i + 2$
$a_k = \lfloor \frac{k}{2} \rfloor$

$> \lfloor \frac{(i+2)}{2} \rfloor$ (By $k > i + 2$)

$= \lfloor \frac{i}{2} + 1 \rfloor$

$= \lfloor \frac{i}{2} \rfloor + 1$

$= \lfloor \frac{i}{2} \rfloor$

$= a_i$ (By the defn of A)

Then $a_k > a_i$
Hence $k > j \rightarrow a_k > a_i$
Since k is an arbitrary natural number, $\forall k \in \mathbb{N}, k > j \rightarrow a_k > a_i$
Since j is a natural number, $\exists j \in \mathbb{N}, \forall k \in \mathbb{N}, k > j \rightarrow a_k > a_i$
Since i is an arbitrary natural number, $\forall i \in \mathbb{N}, \exists j \in \mathbb{N}, \forall k \in \mathbb{N}, k > j \rightarrow a_k > a_i$