CSC165 WINTER 2004
ASSIGNMENT #2

(1) The game of euchre (pronounced YOU-cur) is played with the cards of rank at least 9: 9, 10, J(ack), Q(ueen), K(ing) and A(ce). There are four suits: S(pade), H(eart), D(iamond) and C(lub), and each suit has a card of each rank. The suits also have a colour: S and C are black, H and D are red. We denote a specific card, e.g. the 10 of C, by writing the rank and suit together: 10C. A card is considered to be the colour of its suit.

For each round there is a bidding process that determines one of the four suits as the trump suit. After determining the trump suit, the J of that suit is called the right bower and the other J of the same colour suit is called the left bower. E.g., if S is the trump suit then JS is the right bower and JC is the left bower.

Let $E =$ the set of all (24) cards (in euchre).
Let $H =$the set of all cards in Tia's hand.
Let $T(c) =$"card c's suit is the trump suit".
Let $S(c, d) =$ "cards c and d are the same colour".
Let $J(c) =$ "card c is a Jack".

(a) Write the following sentences symbolically.
   (i) Tia does not have the right bower in her hand.
   (ii) Tia has the left bower in her hand.

(b) The following Java method returns whether card c beats card d. Rewrite it without using "`if`" (and without using its variants like "`? :`" nor "`while`"). Don't use the rules of euchre, instead go purely by the logic in the method.

```
public static boolean beats(Card c, Card d) {
      if (isRightBower(c)) {
          return true;
      } else if (isLeftBower(c)) {
          return !isRightBower(d);
      } else if (!isRightBower(d) && !isLeftBower(d)) {
          if {areSameSuit(c, d)) {
            return isHigherRank(c, d);
          } else {
            return isTrump(c);
          }
      } else {
          return false;
      }
}
```

(2) Express each of the following statements symbolically.
  (a) Some method is called by all methods.
  (b) Every method calls itself.
  (c) Some method calls a method that calls it.
  (d) No method calls another method that calls itself.

(3) Consider these sentences about a sequence of integers $a_0, a_1, a_2, \ldots$:
  (S1) $\forall i \in N, a_i \leq a_{i+1} \leq a_{i+2} \vee a_i \geq a_{i+1} \geq a_{i+2}$.
  (S2) $\forall i \in N, \exists j \in N, \forall k \in N, k \geq j \rightarrow a_i \leq a_k$.
  (S3) $\exists i \in N, \forall j \in N, \exists k \in N, k \geq j \wedge a_i < a_k$.
  And consider these sequences of integers:
  (A) $-1, 0, 1, 0, -1, 0, 1, 0, -1, \ldots$.
  (B) $0, 1 + \sin(\pi/2), 2 + 2\sin(2\pi/2), 3 + 3\sin(3\pi/2), \ldots$.
  (C) $-27, -8, -1, 0, 1, 8, 27, 64, 125, 216, \ldots$.
  (a) Express $x \leq y \leq z$ as a (symbolic) sentence using $\leq$ to compare only *two* elements at a time.
  (b) For each sentence, express its negation, moving the negation inside as much as possible.
  (c) For each sentence, for each sequence, state whether the statement is true for the sequence and give a brief justification (mainly in English). In particular, where an example or counter-example proves your claim be sure to give it.

(4) Let D be some domain and for all $x \in D$ let $p(x)$ and $q(x)$be statements about x. Consider these statements:
  (S4) $\forall x \in D, p(x) \rightarrow q(x)$.
  (S5) $(\forall x \in D, p(x)) \rightarrow (\forall x \in D, q(x))$.
  (S6) $\exists x \in D, p(x) \rightarrow q(x)$.
  (S7) $(\exists x \in D, p(x)) \rightarrow (\exists x \in D, q(x))$.
  For each of the following statements, state whether it's true. Where an example or counter-example database D and predicates p and q justify your claim, give them.
  (a) (S4) implies (S5).
  (b) (S5) implies (S4).
  (c) (S6) implies (S7).
  (d) (S7) implies (S6).