

CSC 165H1 F October Midterm 2004  
Duration — 50 minutes  
Aids allowed: none

Student Number: \_\_\_\_\_

Last Name: \_\_\_\_\_ First Name: \_\_\_\_\_

Lecture Section: L0101 Instructor: Campbell

|                                   |           |         |             |            |
|-----------------------------------|-----------|---------|-------------|------------|
| Tutorial Section:<br>(circle one) | BA-2159   | BA-3008 | BA-3012     | BA-3116    |
|                                   | Nathanael | Midori  | Alex        | Jennifer   |
|                                   | Hyafil    | Hyndman | Levinshtein | Listgarten |

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*Do **not** turn this page until you have received the signal to start.*  
(Please fill out the identification section above,  
and read the instructions below.) *Good Luck!*

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This midterm consists of 5 questions on 6 pages (including this one).  
*When you receive the signal to start, please make sure that your copy is complete.*

If you use any space for rough work, indicate clearly what you want marked.

If you are unable to answer a question (or part of a question), you will get 20% of the marks for the question (or part of the question) if you state clearly that you do not know how to answer. Note that you will *not* get those marks if your answer contains contradictory statements (such as “I do not know how to answer” followed or preceded by parts of a solution that have not been crossed off).

# 1: \_\_\_\_\_/ 8

# 2: \_\_\_\_\_/15

# 3: \_\_\_\_\_/ 6

# 4: \_\_\_\_\_/ 6

# 5: \_\_\_\_\_/10

TOTAL: \_\_\_\_\_/45

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**Question 1.** [8 MARKS]

Consider the following sentence:

(S)  $A$  is true when  $B$  and  $C$  are both true.

**Part (a)** [2 MARKS]

Rewrite (S) using precise symbolic notation.

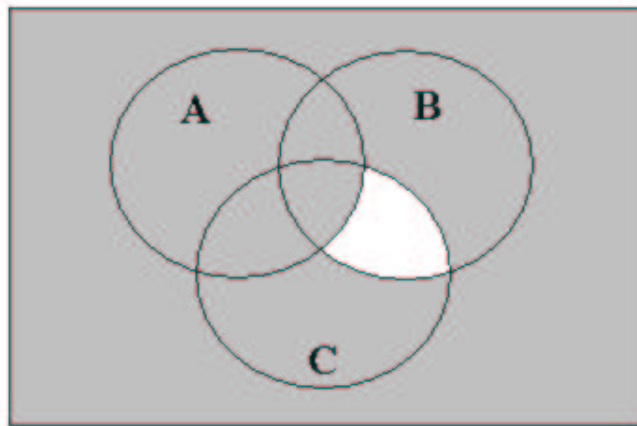
**Solution:**

$$B \wedge C \rightarrow A$$

**Part (b)** [3 MARKS]

Assuming (S) is true, shade the regions of the Venn diagram that may contain elements.

**Solution:**

**Part (c)** [1 MARK]

Assuming (S) is true, what can be concluded about an element if you know that it is a A?

**Solution:**

Nothing.

**Part (d)** [1 MARK]

Assuming (S) is true, what can be concluded about an element if you know that it is not an A?

**Solution:**

It is not both a B and a C. (I.e., It is not a B or it is not C).

**Part (e)** [1 MARK]

Assuming (S) is true, what can be concluded about an element if you know that it is not a C?

**Solution:**

Nothing.

**Question 2.** [15 MARKS]

Let  $P$  represent the set of all people.

Let  $C$  represent the set of all courses.

Let  $e(p, c)$  = person  $p$  is enrolled in course  $c$ .

Let  $i(p, c)$  = person  $p$  is the instructor for course  $c$ .

Using only the domains and predicates defined above, rewrite each of the following sentences using precise symbolic notation.

**Part (a)** [3 MARKS]

Every course has at least one instructor.

**Solution:**

$$\forall c \in C, \exists p \in P, i(p, c)$$

**Part (b)** [3 MARKS]

Some course has exactly one student enrolled.

**Solution:**

$$\exists c \in C, \exists x \in P, e(x, c) \wedge (\forall y \in P, e(y, c) \rightarrow x = y)$$

**Part (c)** [3 MARKS]

No student is enrolled in every course.

**Solution:**

$$\forall p \in P, \exists c \in C, \neg e(p, c)$$

**Part (d)** [3 MARKS]

If a course does not have students enrolled in it, then it does not have an instructor.

**Solution:**

$$\forall c \in C, ((\forall p_1 \in P, \neg e(p_1, c)) \rightarrow \forall p_2 \in P, \neg i(p_2, c))$$

**Part (e)** [3 MARKS]

Some instructor teaches both course “ $c_1$ ” and course “ $c_2$ ”.

**Solution:**

$$\exists p \in P, i(p, c_1) \wedge i(p, c_2)$$

**Question 3.** [6 MARKS]

Rewrite the following Java code without using “if” and without using variants like “?:”, “while”, and “switch”. Simplify your answer (show your work). Your solution should be Java code.

```

if(x && y) {
    if (!z) {
        return z || y;
    } else {
        return false;
    }
} else {
    if (!x || y) {
        return true;
    } else {
        return false;
    }
}

```

**Solution:**

```

return ((x && y) && !z && (z || y)) || (!(x && y) && (!x || y));
return ((x && y) && ((!z && z) || (!z && y))) || (!(x && y) && (!x || y));
return (x && y && !z) || ((!x || !y) && (!x || y));
return (x && y && !z) || !x;

```

**Question 4.** [6 MARKS]

Give the direct proof structure outline for:  $\forall i \in \mathbb{N}, \exists j \in \mathbb{N}, i > j \wedge a_i < a_j$

**Solution:**

Let  $i \in \mathbb{N}$

Let  $j = \underline{\hspace{2cm}}$

Then  $j \in \mathbb{N}$

         [ proof of  $i > j$  ]

         [ proof of  $a_i < a_j$  ]

So  $i > j \wedge a_i < a_j$

Since  $j$  is a natural number,

$\exists j \in \mathbb{N}, i > j \wedge a_i < a_j$

Since  $i$  is an arbitrary natural number,

$\forall i \in \mathbb{N}, \exists j \in \mathbb{N}, i > j \wedge a_i < a_j$

**Question 5.** [10 MARKS]

Consider the following sentences about sequences of integers  $a_0, a_1, a_2, \dots$ :

$$(S1) \forall i \in \mathbb{N}, \exists j \in \mathbb{N}, i \geq j \wedge a_i < a_j$$

$$(S2) \exists j \in \mathbb{N}, \forall i \in \mathbb{N}, i \geq j \rightarrow a_i < a_j$$

**Part (a)** [4 MARKS]

Rewrite each of the sentences above in clear, natural English.

(S1):

**Solution:**

All elements are smaller than some element that appears earlier in the sequence.

(S2):

**Solution:**

Some element is bigger than all elements that appear after it.

**Part (b)** [2 MARKS]

Determine whether each sentence above is true or false for the following sequence and circle the appropriate answer:

1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, ...

(S1) is: TRUE /  FALSE / I DON'T KNOW (FOR 20%)

(S2) is: TRUE /  FALSE / I DON'T KNOW (FOR 20%)

**Part (c)** [2 MARKS]

Determine whether each sentence above is true or false for the following sequence and circle the appropriate answer:

0, 0, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, ...

(S1) is: TRUE /  FALSE / I DON'T KNOW (FOR 20%)

(S2) is: TRUE /  FALSE / I DON'T KNOW (FOR 20%)

**Part (d)** [2 MARKS]

Determine whether each sentence above is true or false for the following sequence and circle the appropriate answer:

5, 4, 5, 3, 4, 3, 4, 2, 3, 2, 3, 1, 0, 0, 0, 0, ...

(S1) is: TRUE /  FALSE / I DON'T KNOW (FOR 20%)

(S2) is: TRUE /  FALSE / I DON'T KNOW (FOR 20%)

Use this page for rough work.

Total Marks = 45

Student #: \_\_\_\_\_

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END OF EXAMINATION