CSC 263 Homework 4

Due August 10, 2004

1. (15 points) Let $G = (V, E)$ be an undirected graph that is connected. Assume that we have run BFS on G starting at some vertex s.

   (a) Show that the following set of edges constitutes a spanning tree of G:

   $$T = \{\{v, p[v]\} \mid v \in V\}.$$

   (b) Let each edge $e = \{u, v\} \in E$ have the following weight:

   $$w(e) = d[u] + d[v].$$

   Is T a minimum cost spanning tree for G, w? Prove it.

2. (15 points)

   (a) Describe how to implement `DECREASE-PRIORITY(v,k)` on a heap.

   (b) Prove that any heap on n nodes has at most $\lceil n/2^{h+1} \rceil$ nodes of height h.

3. (20 points)

   (a) Give the pseudocode for an algorithm that takes an array A of n natural numbers and outputs them in (increasing) sorted order using the priority queue ADT.

   (b) What is the best worst-case running time (in O-notation) you can achieve for this algorithm if you implement the priority queue using a heap? Explain.

   (c) Give the pseudocode for an algorithm that takes an array A of n natural numbers and outputs the median number using the priority queue ADT.

   (d) What is the best worst-case running time (in O-notation) you can achieve for this algorithm if you implement the priority queue using a heap? Explain.

   (e) Assume you have a function `FindMedian(A,n)` that implements part (c) in the amount of time in part (d). Using this function as a *black box* (that is, just calling it without modifying its content) show how to improve the worst-case running time of Quicksort. How fast does it become (in O-notation)? Explain.