CSC 263 Homework 1

Due June 8, 2004

1. (20 points)

   (a) Show that $\log(n!) \in \Theta(n \log n)$.

   Solve the following recurrence relations (show your work):

   (b) $A(n) = 3A(n-1) + 3$, where $A(1) = 1$.

   (c) $A(n) = 3A(n/3) + 3n$, where $A(0) = 6$. You can give your answer in Θ notation.

   (d) $A(n) = 3 + \frac{1}{3}\sum_{i=1}^{n-1} A(i)$, where $A(1) = 1$.

2. (10 points) Recall ListSearch from Lecture 1. For fixed n, let L be the linked list $1 \to 2 \to \ldots \to n$. In lecture, we analyzed the average-case complexity of ListSearch over the sample space

   $$S_n = \{(L, k) \mid 0 \leq k \leq n\},$$

   where each input in S_n was equally likely. Calculate $T_{avg}(n)$, the average-case complexity of Listsearch over S_n where $\Pr(k = 0) = 1/2$ and all other values of k are equally likely.

3. (20 points) Hard disks are (for our purposes) disks that look like a record and spin about their center. They are divided into concentric rings called *tracks*, which are in turn divided into slices called *sectors*. Any item of data is located at a (*track, sector*) pair (see figure). To read this location, the disk-arm executes the following algorithm:

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Read (track x, sector y)
  Move arm from center out to track x
  Wait for beginning of sector y to spin under arm
  Read it while it spins under arm
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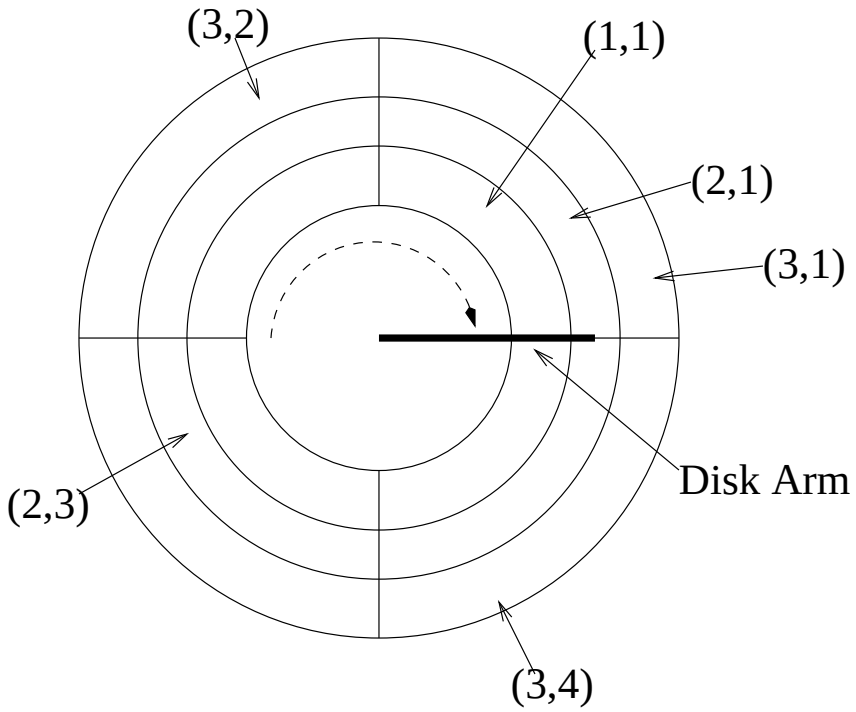
   The tracks are numbered from 1 to n starting with the innermost. Let's say it takes i units of time to move the arm out to track i. Each track has 4 sectors. The disk spins at a rate of 1 revolution per unit time. Assume all (*track, sector*) pairs are equally likely to contain the data we want to read.

   (a) Let $1 \leq i \leq n$ and $1 \leq j \leq 4$. What is $t(i, j)$, the time needed to read sector (i, j)?

   (b) What is the worst-case running time of the algorithm?

   (c) What is $\Pr(i, j)$, the probability of sector (i, j) (we know each sector is equally likely)?

(d) What is the average-case running time of the algorithm?



4. (20 points) **Programming Question:** Coming soon!