Homework Assignment 2: Game Theory

CSC 200Y: Social and Economic Networks

Out: November 14, 2014
Due: December 1, 2014 (note slight extension of due date): in class (by 4:00PM)

Be sure to include your name and student number with your assignment. If your handwriting is *possibly* illegible, be sure to hand in your assignment in some typed form.

1. Consider the following game in matrix form with two players. Payoffs for the row player Izzy are indicated first in each cell, and payoffs for the column player Jack are second.

	X	Y	Z
S	$5, 2$	$10, 6$	$25, 10$
T	$10, 12$	$5, 6$	$0, 0$

   (a) This game has two *pure* strategy Nash equilibria. What are they (justify your answer)? Of the two pure equilibria, which would Izzy prefer? Which would Jack prefer?

   (b) Suppose Izzy plays a *strictly mixed* strategy, where both S and T are chosen with positive probability. With what probability should Izzy choose S and T so that *each* of Jack's three pure strategies is a best response to Izzy's mixed strategy.

   (c) Suppose Jack wants to play a mixed strategy in which he selects X with probability 0.7. With what probability should Jack plays actions Y and Z so *both* of Izzy's pure strategies is a best response to Jack's mixed strategy? Explain your answer.

   (d) Based on your responses above, describe a mixed strategy equilibrium for this game in which both Jack and Izzy play *each* of their actions (pure strategies) with positive probability. Explain why this is in fact a Nash equilibrium (you can rely on the quantities computed in the prior parts of this question).

   (e) If we *swap* two of Izzy's payoffs in this matrix—in other words, if we replace one of his payoffs r in the matrix with another of his payoffs t from the matrix, and replace t with r—we can make one of his strategies dominant. What swap should we make, which strategy becomes dominant, and why is it now dominant?

2. Answer all parts (a) through (e) of Question 15 in Section 6.11 (Chapter 6) of the textbook (found on p.187 of the print version).

3. Suppose a seller runs a second-price, sealed-bid auction for a painting. There are two bidders with independent, private values. The seller does not know their precise valuations, but knows: (a) each bidder i has one of three values, $v_i = 2$, $v_i = 4$ or $v_i = 8$; and (b) each of these values is equally likely (i.e., occurs with probability $\frac{1}{3}$). When running the auction, if the two bids are tied (say, at x), the winner is chosen at random (and pays x). The seller has no value for the painting (i.e., her valuation is 0).

   (a) Assume both bidders use their dominant strategies for bidding in a second-price auction. What is the seller's expected revenue in this auction? Please explain your answer.

   (b) Now the seller decides to set a *reserve price* of r—as discussed in class, this means that if the highest bid is at least r, then the painting will go to the highest bidder, and the winner will pay the maximum of r and the second-highest bid.

   Suppose the reserve price is set to $r = 4$. Assume both bidders use their dominant strategies. What is the seller's expected revenue in this auction? Please explain your answer. If the expected revenue increases or decreases relative to your answer in part (a), give a qualitative explanation for why this change occurs.

   (c) Is there a better reserve price than $r = 4$ (i.e., that will provide more revenue for the seller)? Give a brief justification for your response.

4. A supplier is offering five different manufacturing widgets (W1, W2, W3, W4, W5) for sale to five different companies (companies A, B, C, D, E). The companies each have the following valuations for the different widgets based on how well they serve the companies needs:

Widget	W1	W2	W3	W4	W5
A	16	8	8	8	8
B	17	8	8	10	20
C	18	8	10	24	22
D	19	10	27	25	24
E	20	32	28	26	24

However, each company only needs *one* widget (so having two or more widgets is only as valuable as having the most valuable of those widgets; that is, they are "partial subsititutes").

The supplier decides to run a sequence of five *second-price auctions*, selling each of the widgets in turn. We assume each auction proceeds as follows:

- In the first auction, all five companies bid. The winning company receives the widget being auctioned, then "leaves," that is, it participates in none of the subsequent auctions. The remaining four companies participate in the second auction, and again the winner receives the widget and "leaves." And so on. So five companies bid in the first auction, four companines bid in the second auction, three companines in the third, two in the fourth, and one in the fifth.
- Each company participating in an auction bids using its dominant strategy for the second-price auction. In other words, it behaves in any auction—using its valuation for the widget being sold—as if it believed their would be no future opportunity to obtain a different widget. So they will act based on our understanding of second-price auctions for single items. (We sometimes call this "myopic" bidding.)
- The supplier has a valuation of zero for all widgets.

The supplier must now decide in which order to auction the items.

(a) Suppose the supplier auctions the items in the following order: W1, W2, W3, W4, W5. For each of the five auctions (call them A1, A2, etc.), state:

- who participates in the auction;
- what each participant bids;
- who wins the auction;
- the winner's value for the widget;
- the price paid for the widget;

What is the *total social welfare* created through this sequence of auctions? What is the *total revenue* received by the supplier?

(b) If you knew the companies' valuations, in what order would you recommend that the supplier auction the widgets so as to *maximize social welfare*? You don't have to justify the choice of the ordering; but given your ordering, describe the participants, bids, winner and price for each of the five auctions as in part (a).

What is the total social welfare created through this sequence of auctions? What is the total revenue received by the supplier?

(c) If you knew the companies' valuations, in what order would you recommend that the supplier auction the widgets so as to *maximize revenue received*? You don't have to justify the choice of the ordering; but given your ordering, describe the participants, bids, winner and price for each of the five auctions as in part (a).

What is the total social welfare created through this sequence of auctions? What is the total revenue received by the supplier?

(d) We assumed above that each company, once it receives a widget, leaves the auctioning process. Now suppose, instead, that a company that has obtained one widget will participate in a *future* auction if the widget being sold has greater value than the one they currently own. How should a company who owns a widget bid in an auction for a widget that is more valuable than the one they currently own? (As above, assume they are "myopic" in the sense that they do not anticipate further widgets being sold later.)

Returning to the original ordering in part (a), how would this behavior impact the final allocation of widgets to companies? What would the resulting social welfare be? Justify your response.

(e) Suppose instead of having value for one widget, companies A and B, being rather large, can actually each derive a value from two (specific) widgets that is greater than the sum of their individual values. Specifically,

- If A wins W1 and W4, its value is the sum of the individual values of these widgets *plus* 10 (i.e., the widgets offer some synergies, so their value in combination is 10 greater than the sum of their individual values).
- If B wins W2 and W3, its value is the sum of the individual values of these widgets *plus* 10.

If A or B win any single widget, the values are as above. The other three companies also have values as above.

Suppose the supplier knows that A and B value these specific pairs of widgets more than the individual widgets that make up the pairs. He also knows that all other widgets have independent valuations and are partial subsitutes (and all widgets are partial substitutes for the other three companies). The supplier does not know the precise valuations of the individual widgets or the pairs (for any of the five companies). But suppose the supplier has a "rough" idea of the valuations in the form of a prior distribution over values that assigns each buyer a uniform probability of having a value within some small range (say, plus or minus 5) of the actual valuations given. (The details aren't important.)

The supplier is allowed to run a sequence of second-price auctions, but is allowed to bundle items together run an auction for a bundle as well as for individual items. What sequence of second-price auctions would you advise (what bundles, if any, should be sold; what items should be sold individually; what ordering should you use; should a reserve price be used)? Give a qualitative justification (not a precise quantitative one) for your suggested approach and the tradeoffs that you considered. *This question is intentionally open-ended. There is no right answer, just more or less "thoughtful" answers.*

5. Recall the *MLA game* as described on Slide 4 of Lecture 9 with its equilibria described on slides 6 and 11. This game involves three members of a legislative assembly (three MLAs) deciding how to vote on a pay raise.

   Suppose now that each of the MLAs votes in *sequence*, using some pre-determined order (e.g., alphabetical). Let's call the MLAs 1, 2 and 3: MLA 1 votes first; MLA 2 votes next knowing exactly how MLA 1 voted; and finally MLA 3 votes knowing the two previous votes.

   (a) Draw the extensive form game tree for this sequential voting game. How will each MLA vote assuming that a subgame perfect equilibrium is played? What is the payoff for each of the MLAs? Explain.

   (b) Describe one profile of Nash equilibrium strategies for this extensive form game that is *not* subgame perfect.

   (c) Now suppose that the MLAs have seen a recent poll, which could potentially influence the payoff of MLA 3. Specifically, the new poll indicates a level of fiscal conservatism in the district of MLA 3 that makes it uncertain how voters will react if she votes for a pay raise. As such MLA 3's payoffs are now random variables. She will realize her original payoffs with probability 0.4; however, with probability 0.6 she will receive the following payoffs:

   - MLA 3's revised payoffs:
     - if the raise passes, and MLA 3 votes for: -1
     - if the raise passes, and MLA 3 votes against: 2
     - if the raise fails, and MLA 3 votes for: -3
     - if the raise fails, and MLA 3 votes against: 0

   In other words, there is a 60% chance that her payoff when voting for a raise will be reduced by 2. All three MLAs are aware of this poll and its implications.

   Notice now that the payoffs for MLA 3 are random events. What is the matrix form of the game in which all three MLAs vote simultaneously, using the *expected* payoffs of MLA 3?

   Draw the extensive form game tree for this new game assuming they vote sequentially (in the same order as in part (a)). How will each MLA vote assuming that a subgame perfect equilibrium is played? What is the payoff for each of the MLAs? Explain.