Definition

1. For any integer $k \leqslant n$, let $U_k(s, e)$ be the set of jobs whose index is lower than or equal to k and whose release date is in the interval $[s, e)$, i.e. $U_k(s, e) = \{J_i | i \leqslant k \text{ and } s \leqslant r_i < e\}$.
2. Let $W_k(s, e)$ be the maximal weight of a subset of $U_k(s, e)$ such that there is a feasible schedule S of these jobs such that

   (i) S is idle before time $s + p$,
   (ii) S is idle after time e,
   (iii) starting times of jobs on S belong to Θ.

   Notice that if the subset of $U_k(s, e)$ is empty, $W_k(s, e)$ is equal to 0.

Proposition 2. (cf. Figure 1). *For any value of k in* $[1, n]$ *and for any values s, e with* $s \leqslant e$, $W_k(s, e)$ *is equal to* $W_{k-1}(s, e)$ *if* $r_k \notin [s, e)$ *and to the following expression otherwise*:

$$\max\left(W_{k-1}(s, e), \max_{\substack{s' \in \Theta \\ \max(r_k, s+p) \leqslant s' \leqslant \min(d_k, e) - p}} (w_k + W_{k-1}(s, s') + W_{k-1}(s', e))\right)$$

Proof. Let W' be the expression above. If $r_k \notin [s, e)$, the result obviously holds since $U_k(s, e) = U_{k-1}(s, e)$. We now consider the case where $r_k \in [s, e)$.

We first prove that $W' \leqslant W_k(s, e)$.

1. If $W' = W_{k-1}(s, e)$ *then* $W' = W_{k-1}(s, e) \leqslant W_k(s, e)$.
2. If there is a value s' in Θ such that $\max(r_k, s + p) \leqslant s \leqslant \min(d_k, e)\text{-}p$ and such that $W' = w_k + W_{k-1}(s, s') + W_{k-1}(s, e)$, let X and Y be two subsets of the set of jobs P that realize, respectively, $W_{k-1}(s, s')$ and $W_{k-1}(s', e)$. Because of the definition of W, the sets X and Y are disjoint and moreover, there exists a feasible schedule of X that fits in $[s + p, s']$ and there exists a feasible schedule of Y that fits in $[s' + p, e]$. Thus, $X \cup Y \cup \{J_k\}$ is a set whose weight is W' and there is a schedule of the jobs in this set that does not start before $s + p$ and that ends before e (take the schedule of X, schedule J_k at time s' and add the schedule of Y, see Figure 1). On top of that, starting times belong to Θ. Hence, $W' \leqslant W_k(s, e)$.

We now prove that $W_k(s, e) \leqslant W'$.

Let Z be a subset that realizes $W_k(s, e)$. If J_k does not belong to Z then $W_k(s, e) = W_{k-1}(s, e) \leqslant W'$. Now suppose that J_k belongs to Z. According to the definition of $W_k(s, e)$, there is a schedule S of Z that fits in $[s + p, e]$ on which starting times belong to Θ.

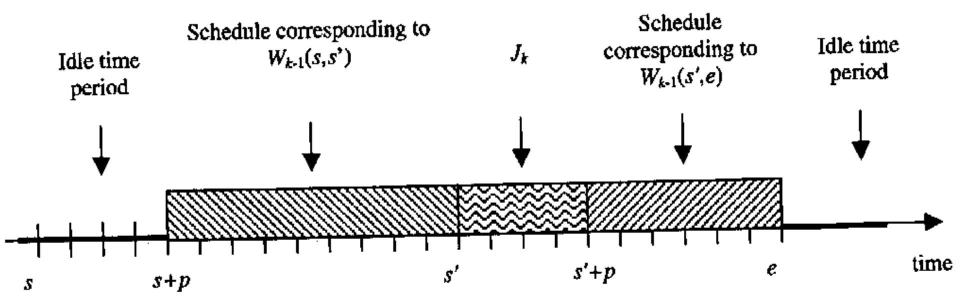

Figure 1. Illustration of Proposition 2