Revised Due Date: Monday, December 6, beginning of lecture

NOTE: Each problem set only counts 5% of your mark, but it is important to do your own work (but see below). Similar questions will appear on the first term test. You may consult with others concerning the general approach for solving problems on assignments, but you must write up all solutions entirely on your own. Anything else is *plagiarism*, and is subject to the University's Code of Behavior. You will receive 1/5 points for any (non bonus) question/subquestion for which you say "I do not know how to answer this question". You will receive .5/5 points if you just leave the question blank.

1. (20 points)

   Consider the makepsan problem for the restricted machines problem when all processing times are identical; that is, a job $J_j = (p_j, S_j)$ with $p_j = 1$ for $1 \leq j \leq n$ and $S_j \subseteq \{1, \ldots, m\}$. Indicate how binary search and max flow can be used to obtain (in polynomial time) an optimal assignment of jobs to their allowable machines. Hint: recall how we used flows to solve max matching in bipartite graphs.

2. (20 points)

   Consider the following weighted *4 to 2 frequency set cover problem*: We are given a collection of sets $\mathcal{S} = \{S_1, \ldots, S_m\}$ for $S_i \subseteq U$ with the property that every $u \in U$ occurs in exactly four different sets S_i in $\mathcal{S}$. There is also a cost function $w : \mathcal{S} \rightarrow \mathbb{R}^+$ and we let w_i denote the cost of set S_i. A feasible solution is a sub-collection $\mathcal{S}' \subseteq \mathcal{S}$ such that every $u \in U$ occurs in at least two different sets S_i in $\mathcal{S}'$. The goal is to find a feasible solution $\mathcal{S}'$ so as to minimize the cost $w(\mathcal{S}') = \sum_{i:S_i \in \mathcal{S}'} w_i$.

   (a) Formulate the *4 to 2 frequency set cover problem* as an $\{0, 1\}$ IP

   (b) Show how to use LP relaxation + rounding to obtain an 3-approximation algorithm. Explain why your rounded solution is a feasible solution to the IP and why it provides a 3-approximation.

3. (20 points)

   Consider the following call routing problem. There is an n node tem Show how to use LP relaxation + rounding to obtain an 3-approximation algorithm. Explain why your rounded solution is a feasible solution to the IP and why it provides a 3-approximation. bi-directional ring network $G = (V, E)$ upon which calls must be routed. That is $V = \{0, 1, \ldots, n-1\}$ and $E = \{(i, i+1 \; mod \; n)\} \cup \{(i, i-1 \; mod \; n)\}$ and calls c_j are pairs (s_j, f_j) originating at node s_j and terminating at node f_j. Each call can be routed in a clockwise or counter-clockwise direction. The load L_e on any directed edge is the maximum number of calls routed on this edge. The goal is to minimize $max_{e \in E} L_e$.

   (a) Formulate this problem as an IP.
   Hint: Consider variables x_j and y_j that indicate the direction of call c_j. (You can also use just one indicator variable to represent the direction but I think it might be easier to think in terms of two such variables.)

(b) Using an LP relaxation of this problem, show how to derive a 2-approximation algorithm.

4. (20 points)

The weighted max cut problem is defined as follows: Given an edge weighted graph $G = (V, E, w)$ with $w : E \rightarrow \mathbb{R}^+$, the goal is to find a cut (A, B) so as to maximize the weight of the cut, defined as $\sum_{(u,v)\in E, u\in A, v\in B} w(u, v)$.

Consider the following randomized algorithm for the weighted max cut problem:
For each $v \in V$
    With equal probability (i.e. $\frac{1}{2}$) decide to place v in A or B.
End For

(a) What is the expected weight of the cut generated by this randomized algorithm, in terms of $W = \sum_{(u,v)\in E} w(u, v)$? Briefly justify your claim.

(b) Use the method of conditional expectations to de-randomize the randomized algorithm.

(c) How would you state the de-randomized algorithm as a greedy algorithm without any mention of probabilities?