

## CSC373S Lecture 16

- For most of today's class we discussed the problem set.
- We will now begin discussing network flows. (See link to "old lecture notes")

Let us call a graph  $G = (V, E)$  "bi-directional" if  $(u, v) \in E$  implies  $(v, u) \in E$ . (In the notes we say "bidirected" but apparently that has another meaning in graph theory.)

A flow network  $\mathcal{F} = (G, c, s, t)$  consists of a bidirectional graph  $G$  with an edge capacity function  $c : E \rightarrow \mathbb{R}^{\geq 0}$  and a distinguished source node  $s$  and terminal node  $t$ .

As the notes say it would be better to call this a capacitated network but flow network is standard. Note: we will only be considering a single commodity "splittable flow" from a single source to a single terminal. There are many other flow problems. Multi-commodity flows are a subject of wide interest.

Intuitively we want to transport as much "material" (eg oil) from  $s$  to  $t$  but we (that is, the flow) cannot exceed the capacity of any edge in the network and we cannot store any material at a node. The latter condition implies that "flow in" of material into a node must equal "flow out" except for the source which just ships out material and the terminal which just receives material.

While we can directly formalize this intuitive "flow in = flow out" concept (and this is what the KT text does), we choose to follow CLRS and our old lecture notes in using an equivalent formalization which at first may seem a little strange but actually makes the development much more mathematically elegant.