CSC373F **Term Test 2 Solutions** **Fall, 2017**

Last name: ________________ First name: ________________
Markus login:

AIDS ALLOWED: One page (two sides) of handwritten notes
Answer ALL questions on test paper. Use backs of sheets for additional space. For every question, briefly justify your answer. You can use your knowledge from lectures, the texts, or the problem set as part of any answer.

REMINDER: You get 20% for any question or subquestion if you state "I do not know how to answer this question". You get 10% for any question which you just leave blank.

1. (15 points) Suppose we have a maximum integral flow f with $val(f) \geq 2$ in a flow network $\mathcal{F} = (G, s, t, c)$ with unit capacities (i.e. $c(e) = 1$ for all edges e in the network. For each of the following, indicate if the statement is true or false and give a one or two sentence justification for your answer.

   - There is necessarily an edge e for which increasing the capacity of e by one unit will result in an increase of exactly one unit in $val(f)$.

     Solution False. There can be several disjoint min cuts.

   - There is an edge e such that decreasing the capacity of e by two units will result in a decrease of exactly two units in $val(f)$.

     Solution False. This is false for a trivial reason since the question stated that all capacities are unit and capacities must be non-negative in a flow newtwork. A better question would have been to say that all capacities are integral. If we had said integral and not unit, the answer would be True since we can then decrease the capacity of an edge in the min cut by two units.

   - Given an integer k, there is a polynomial time (i.e. polynomial in k and the description of the flow network) algorithm that will determine whether or not there are at least k edge disjoint paths from s to t.

     Solution True. Set all capacities to 1 and see if the max flow is equal to k.

2. (15 points) *Assume* (even though we may not believe it is true) that $P = NP$.

- Given the above assumption, does it follow that the following optimization problem can be solved optimally in polynomial time:

  Given: A 3CNF formula $F = C_1 \wedge C_2 \ldots \wedge C_m$.
  Output: A truth assignment τ^* to the propositional variables that maximizes the number of satisfied clauses. That is,
  $(\sum_i C_i$ satisfied by $\tau^*) \geq (\sum_i C_i$ satisfied by $\tau)$ for any truth assignment τ. If your answer is NO, briefly explain why; if your answer is YES , briefly explain how you would find the desired τ^*.

  Solution If $P = NP$, then yes the problem can be solved in ploynomial time. Here is how we can do it. First, we observe that the decision problem for $L = \{(F, k) : F$ is a 3CNF formula for which there is a truth assignment τ satisfying at least k clauses$\}$ is in the class NP. Hence if $P = NP$, then we can solve this decision problem in polynomial time. Using this decision problem we can determine the maximum number k^* of clauses that can be satisfied by some truth assignment τ^*.

  Now knowing k^*, we need to determine an appropriate truth assignment τ^*. We will do so, one prositional variable at a time. Sat the variables are $x_1, \ldots, x_n$. We substitute $x_1 = true$ in F; let $F_1 = F|x_1 = true$. We then use the decision procedure to determine if (F_1, k^*) is in L; that is, if F_1 has a truth assignment satisfying k^* clauses. If Yes, then we set $x_1 = true$ and continue with finding the true assignments for $x_2, \ldots, x_n$ with respect to F_1. Otherwise we set $x_1 = false$ and $F_1 = F|_{x1=false}$ and continue.

  This explanation above is a little wordy but the idea is to determine the truth assignment for one variable at a time and then set that variable and continue with the new formula.

3. (15 points)

- (10 points) Let $4SAT$ be the decision problem for satisfiability of CNF formulas with at most 4 literals per clause. Show that $4SAT \leq_p 3SAT$; that is, $4SAT$ transforms to $3SAT$.
  The propositional formula $x \leftrightarrow y$ means that x is true if and only if y is true.
  Hint: Note then that $x \leftrightarrow (y_1 \vee y_2)$ is equivalent to the 3CNF formula $(\bar{x} \vee y_1 \vee y_2) \wedge (\bar{y}_1 \vee x) \wedge (\bar{y}_2 \vee x)$

  Solution The idea is that we need to convert every clause with 4 literals to a set of clasues each having 3 literals. Suppose, $C = x_1 \vee x_2 \vee y_1 \vee y_2$. We then introduce a new variable x and make it equivalent to $y_1 \vee y_2$ (using the indicated clauses) and C is then replaced by $x_1 \vee x_2 \vee x$ and these three additional clauses that make x equivalent to $y_1 \vee y_2$.

  Another perhaps easier transformation is to transform $C = x_1 \vee x_2 \vee y_1 \vee y_2$ to $(x_1 \vee x_2 \vee z) \wedge (y_1 \vee y_2 \vee \bar{z})$.

- (5 points) Show $SAT \leq_p 3SAT$ where SAT is the satisfiabilty question for CNF formulas.

  Solution We can use the above idea to transform any formula with say a maximum of $t \geq 4$ literals per clause to one having at most $t - 1$ literalas per clause. Then by the fact that tranformations are transitive, we will eventually transform to 3SAT.