# Due: Monday, December 4, 2017, 10AM on MarkUs

**You will receive 20% of the points for any (sub)problem for which you write "I do not know how to answer this question." You will receive 10% if you leave a question blank. If instead you submit irrelevant or erroneous answers you will receive 0 points. You may receive partial credit for the work that is clearly "on the right track" .**

1. (5 points)

   In the same way that we can express LPs in standard form, we can express an IP and its dual in standard form. Show that weak duality holds for integer programs in standard form.

2. (25 points)

   Consider the following primal IP and LP in standard form:

   $$\begin{array}{ll} \text{Maximize} & y \\ \text{Subject to} & 2y - 3x \leq 0 \\ & 2y + 3x \leq 3 \\ & y, x \geq 0 \end{array}$$

   For the IP, add the constraints that $x$ and $y$ are integral.
   Draw the feasible LP region. Note: You do not need to submit this with the assignment but it will be helpful to draw the feasible region.

   - (5 points)
     What are the vertices of the feasible region of the primal LP?
   - (5 points) What is the optimal fractional solution?
   - (5 points) What is the optimal integral solution?
   - (5 points) Provide the dual IP of the primal IP.
   - (5 points) Use this example to show that strong duality does not necessarily hold for integer programming.

3. (20 points) Consider the following weighted partial vertex cover problem (WPVC):

   Given: A graph $G = (V, E)$ with vertex weights $w : V \to \mathbb{R}^{\geq 0}$ and edge costs $c : E \to \mathbb{R}^{\geq 0}$. In the standard vertex cover problem, we need to choose a subset $V' \subseteq V$ so that every edge $e = (u, v)$ has to be covered in the sense that at least one of its endpoints $u$ or $v$ must be in the cover $V'$. In the WPVC problem, not every edge has to be covered but an uncovered edge $e$ costs $c(e)$. The objective it to minimize the weights of the partial cover plus the costs of uncovered edges.

   - (10 points) Provide a $\{0, 1\}$ IP for the WPVC problem.
   - (10 points) Provide an approximation guarantee (i.e. approximation ratio) for WPVC by the IP/LP rounding method.

4. (20 points) Consider a model of computation where arithmetic operations take $O(1)$ time and choosing a random number in the range $\{1, 2, \ldots, 2n\}$ takes time $O(1)$. Let $A, B$ and $C$ be $n \times n$ matrices over the ring of integers. We want to verify (with some error probability) that $AB = C$ using $O(n^2)$ arithmetic operations.

- (10 points) Provide a randomized algorithm $ALG$ satisfying the following:
  (a) If $AB = C$, $ALG$ will always say YES
  (b) If $AB \neq C$, $ALG$ will say YES with some constant error probability $p < 1$. i.e. will say NO with probability $1 - p$.
- (10 points) Show how you can verify $AB = C$ with error probability $\frac{1}{n}$. What is the time complexity of your algorithm?

5. (10 points) Consider the exact Max-3-Sat problem:
   Given a propositional CNF formula $F = C_1 \wedge C_2 \wedge \ldots \wedge C_m$ where each clause $C_i$ has exactly 3 literals.
   The objective is to compute a truth assignment so as to maximize the number of satisfied clauses.

   Provide a polynomial time randomized algorithm that in expectation will satisfy a $\frac{7}{8}$ fraction of the clauses.