"Some partial solutions for problem set 2".

- Q2

  We can construct a divide and conquer algorithm for $n \times n$ matrix multiplication based on 3×3 matrix multiplication as follows. Suppose we can multiply two 3×3 matrices in q multiplcations without using commutativity. Say that such a method uses a additions. We will assume for simplicity that $n = 3^k$ for some k. (How would you argue that the asymptotic complexity is not effected by such an assumption?)

  Now we view a $3^k \times 3^k$ matrix A as a 3×3 matrix whose entries are $3^{k-1} \times 3^{k-1}$ matrices. That is ,

  $$A = \begin{matrix} A_{11}A_{12}A_{13} \\ A_{21}A_{22}A_{23} \\ A_{31}A_{32}A_{33} \end{matrix}$$

  Then using the 3×3 method, we can multiply $A \cdot B$ using q matrix multiplications of dimension 3^{k-1} and a additions of such matrices. Matrix addition only costs $O(n^2)$ so the recurrence is $T(n) = qT(n/3) + \theta(n^2)$. For $q > 9$ this recurrence can be seen to be asymptotic to $n^{\log_3(q)}$. So we need $q \leq 21$ so as to beat $n^{\log_2(7)}$.

- Q3

  The idea is to construct a semantic array OPT[i,j,k] = max profit of a subset of intervals $S \subseteq \{1, \ldots, i\}$: machine one ends at time $\leq f_j$ and machine two ends at time $\leq f_k$ with say $i \geq j > k$. This is a 3 dinensional array and one suspects that from the one machine case that we should be able to get away with two dimensions. Indeed this can be done with a little care, say (for example) redefining OPT[i,j] = max profit of a subset of intervals $S \subseteq \{1, \ldots, i\}$: machine one ends with interval I_i and machine two ends with interval I_j. Doing this right does require a little care whereas the three dimensional solution is easy to specify. One also has to search the entire array to find the optimal value but still this would be more efficient as it is a $\Theta(n^2)$ algorithm rather than $\Theta(n^3)$. . More generally, one can solve the m machine case by a DP with complexity $\Theta(n^m)$. More efficient (non DP) methods are known with complexity $\Theta(n^2(n - m)$.