CSC364S (MWF2 section) **Term Test 2; March 5** **Spring, 2004**

Name ______________________ Student No. ______________________

Tutorial: MP202 ____________ UC144 ____________ UC52 ____________

AIDS ALLOWED: One page (two sides) of handwritten notes
Answer ALL questions on test paper. Use backs of sheets for additional space.
Total Marks: 70

REMINDER: You get 20% of any question or subquestion if you state "I do not know how to answer this question". You get 10% of any question which you just leave blank.

1. (a) Let $\mathcal{F} = (G, c, s, t)$ be a flow network in which the capacity function c is an odd valued integer function (i.e. $c(e) = 2k_e + 1$ for some non negative integer k_e). Prove or disprove that the maximum flow is always (i.e. for all such networks) an odd valued integer. [10 points]

(b) Let $\mathcal{F} = (G, c, s, t)$ be a flow network in which the capacity function is integer valued. Suppose that someone has already computed a maximum flow $f : E \to \mathbb{N}$. Now you are being asked to *decrease* the capacity of some edges so as to decrease the max flow by 1. Describe an algorithm which will indicate how to decrease the capacity of the fewest edges (by one unit) so as to achieve this decrease in flow. What is the complexity of your algorithm in terms of $|E|$ and $|V|$ where $G = (V, E)$. [20 points]

2. Suppose $L_1 \leq_p L_2$ and $L_2 \in \mathbf{NP}$. Show $L_1 \circ L_2 \in \mathbf{NP}$ where the concatenation of two languages is defined by:
$L_1 \circ L_2 = \{x | x = x_1 x_2;\text{ for some } x_1 \in L_1, x_2 \in L_2\}$

Hint: What is a certificate and verification predicate for showing $x \in L_1 \circ L_2$ in terms of certificates and verification predicates for membership in L_1 and membership in L_2? [20 points]

3. Consider the max clique optimization problem $CLIQUEOPT$ and the decision problem $CLIQUE$. Show $CLIQUEOPT$ is polynomial time reducible to $CLIQUE$. Definitions: A clique in a graph $G = (V, E)$ is a subset $V' \subset V$ such that for all $u \neq v \in V'$, $(u, v) \in E$. $CLIQUEOPT$ takes a representation of a graph $< G >$ and returns a maximum size clique in the graph and $CLIQUE = \{< G, m > | G$ has a clique of size $m\}$. [20 points]