1. (a) Let $\mathcal{F} = (G, c, s, t)$ be a flow network in which the capacity function $c$ is an odd valued integer function (i.e. $c(e) = 2k_e + 1$ for some non-negative integer $k_e$). Prove or disprove that the maximum flow is always (i.e. for all such networks) an odd valued integer. [10 points]
(b) Let $\mathcal{F} = (G, c, s, t)$ be a flow network in which the capacity function is integer valued. Suppose that someone has already computed a maximum flow $f : E \rightarrow \mathbb{N}$. Now you are being asked to decrease the capacity of some edges so as to decrease the max flow by 1. Describe an algorithm which will indicate how to decrease the capacity of the fewest edges (by one unit) so as to achieve this decrease in flow. What is the complexity of your algorithm in terms of $|E|$ and $|V|$ where $G = (V, E)$.

[20 points]
2. Suppose $L_1 \leq_p L_2$ and $L_2 \in \textbf{NP}$. Show $L_1 \circ L_2 \in \textbf{NP}$ where the concatenation of two languages is defined by:

$L_1 \circ L_2 = \{x | x = x_1x_2; \text{for some } x_1 \in L_1, x_2 \in L_2\}$

Hint: What is a certificate and verification predicate for showing $x \in L_1 \circ L_2$ in terms of certificates and verification predicates for membership in $L_1$ and membership in $L_2$? [20 points]
3. Consider the max clique optimization problem $CLIQUEOPT$ and the decision problem $CLIQUE$. Show $CLIQUEOPT$ is polynomial time reducible to $CLIQUE$. Definitions: A clique in a graph $G = (V, E)$ is a subset $V' \subset V$ such that for all $u \neq v \in V'$, $(u, v) \in E$. $CLIQUEOPT$ takes a representation of a graph $< G >$ and returns a maximum size clique in the graph and $CLIQUE = \{ < G, m > | G$ has a clique of size $m \}$. [20 points]