**CSC364S (MWF2 section)** **Term Test 1; January 30** **Spring, 2004**

Name ______________________________ Student No. ______________________________

Tutorial: MP202 ________________ UC144 ________________ UC52 ________________

AIDS ALLOWED: One page (two sides) of handwritten notes
Answer ALL questions on test paper. Use backs of sheets for additional space.
Total Marks: 50

REMINDER: You get 20% of any question or subquestion if you state “I do not know how to answer this question”. You get 10% of any question which you just leave blank.

1. This problem concerns Kruskals greedy algorithm for the MST problem. Let $G = (V, E)$ be a connected graph and $c : E \rightarrow \Re$ a cost function on the edges.

   (a) Suppose $E' \subset E$ is an acyclic set of edges; that is, the graph $(V, E')$ is acyclic. Indicate how Kruskal's greedy algorithm can be modified so as to compute a minimum spanning tree containing $E'$. How would you define a “promising partial solution”? [10 points]

(b) Suppose that the edges have been sorted and that $c(e_1) = c(e_2) < e_3 \ldots < e_m$. That is, the two least costly edges have the same cost and all other edges have distinct costs. Use the analysis of Kruskals algorithm to show that there is a unique minimum spanning tree. [10 points]

(c) Now suppose that the edges have been sorted and that $c(e_1) = c(e_2) = c(e_3) < e_4 \ldots < e_m$. That is, the three least costly edges have the same cost and all other edges have distinct costs. Provide a graph $G = (V, E)$ with the edge costs as stated so that there are at least two different minimum spanning trees. [5 points]

2. Consider the following variant of the knapsack problem. The input is a set of $N$ items with weights $w_1, \ldots, w_N$, gains $g_1, \ldots, g_n$ and volumes $v_1, \ldots, v_N$; a weight bound $C$, and a volume bound $D$. A subset $S \subseteq \{1, \ldots, N\}$ is feasible if $\sum_{i \in S} w_i \leq C$ and $\sum_{i \in S} v_i \leq D$. The goal is to find a feasible subset $S$ having maximum gain $g(S) = \sum_{i \in S} g_i$. Assume all input parameters are positive integers.

   Outline a dynamic programming solution with complexity $O(NCD)$ for this problem. More specifically

   (a) Define an appropriate semantic array $A$ and show how to compute the optimal value from $A$. [10 points]

   (b) Give an equivalent recursively defined computational array $\tilde{A}$. (Don't forget the base case.) [10 points]

   (c) Intuitively justify why $A = \tilde{A}$ and briefly say how you would prove this equality. [5 points]