Due: Wednesday, March 31, beginning of lecture

NOTE: Each problem set only counts 5% of your mark, but it is important to do your own work. These assignments will be followed by term tests, each worth 15% of your final grade. You may consult with others concerning the general approach for solving problems on assignments, but you must write up all solutions entirely on your own. You may chose to work in pairs and then submit one assignment. Anything else is *plagiarism*, and is subject to the University's Code of Behavior. You will receive 1/5 points for any question/subquestion for which you say "I do not know how to answer this question". You will receive .5/5 points if you just leave the question blank.

1. (a) Let $BIG-VC = \{< G > |G = (V,E)$ and G has a vertex cover of size $|V|-10\}$. Show that $BIG-VC$ is in P.

   (b) Let $HALF-VC = \{< G > |G = (V,E)$ and G has a vertex cover of size $\lceil |V|/2 \rceil\}$. Show that $HALF-VC$ is NP-complete.

2. Consider the following scheduling problem. A job $J_i =< r_i, d_i, p_i >$ is described by three positive integer variables, r_i the release time, d_i the deadline, and p_i the processing time. We are given N such jobs and want to find the largest number of jobs that can be scheduled in a feasible schedule $\sigma : \{1, 2, \ldots, N\} \to \{0, 1, \ldots, D\}$ where D is the latest time that any job can begin and still complete within its deadline and $\sigma() \neq 0$ indicates where to start the i^{th} job (if scheduled). If $\sigma(i) = 0$ then the i^{th} job is not scheduled. Otherwise, in a feasible schedule, $r_i \leq \sigma(i) \leq d_i - p_i$ and for any two scheduled jobs $i \neq j$, $[\sigma(i), \sigma(i)+p_i) \cap [\sigma(j), \sigma(j)+p_j) = \emptyset$. Show that the language $L = \{< J_1, \ldots, J_N \}|$ all N jobs can be scheduled$\}$ is NP-complete.

3. Prove Lemma 9 in the lecture notes. That is, show that neither INF or $\overline{INF}$ is semi-decidable.

4. Consider the language $L = \{< M > |\exists x$ such that M accepts the three strings $x, x0$, and $x1\} \subseteq \{0,1\}^*$. Show that L is semi-decideable but not decideable.

5. Possibly more to follow.