CSC364S (MWF2 section) **Problem Set 2** **Spring, 2004**

Due: Wednesday, March 3, beginning of lecture

NOTE: Each problem set only counts 5% of your mark, but it is important to do your own work. These assignments will be followed by term tests, each worth 15% of your final grade. You may consult with others concerning the general approach for solving problems on assignments, but you must write up all solutions entirely on your own. You may chose to work in pairs and then submit one assignment. Anything else is *plagiarism*, and is subject to the University's Code of Behavior. You will receive 1/5 points for any question/subquestion for which you say "I do not know how to answer this question". You will receive .5/5 points if you just leave the question blank.

1. (a) Let $\mathcal{F} = (G, c, s, t)$ be a flow network in which the capacity function c is an even valued integer function (i.e. $c(e) = 2k_e$ for some non negative integer k_e). Prove that the maximum flow is an even valued integer.

   (b) Let $\mathcal{F} = (G, c, s, t)$ be a flow network in which the capacity function is integer valued. Suppose that someone has already computed a maximum flow $f : E \rightarrow N$. Now you are being asked to increase the capacity of some edges so as to increase the max flow by 1. Describe an algorithm which will indicate how to increase the capacity of the fewest edges (by one unit) so as to achive this increase in flow. What is the complexity of your algorithm in terms of $|E|$ and $|V|$ where $G = (V, E)$.

2. Let $\mathcal{F} = (G, c, s, t)$ be a flow network in which the capacity function is integer valued and $c(e) \leq B$ for all edges e. Describe a "reasonably efficient" encoding of $\mathcal{F}$ as a string over some finite alphabet Σ. What is the length of your encoding as a function of $N = |V|, M = |E|$ and B where $G = (V, E)$?

3. Describe a Turing machine that takes a string $x \in \{a, b\}^*$ and computes its reverse x^R. Your Turing machine should leave x^R and only x^R on the tape upon halting. (You can use a table description as in the notes or a state diagram as depicted in lecture.)

4. Suppose $L_1 \leq_p L_2$ and $L2 \in \mathbf{NP}$. Show $L_1 \cup L_2 \in \mathbf{NP}$.

5. Consider the vertex cover optimization $VCOPT$ and decision VC problems. Show $VCOPT$ is polynomial time reducible to VC.