CSC304: Lecture 6

- Today

Guest lecture by Omer Lev

The topic today is games in extrensive form.

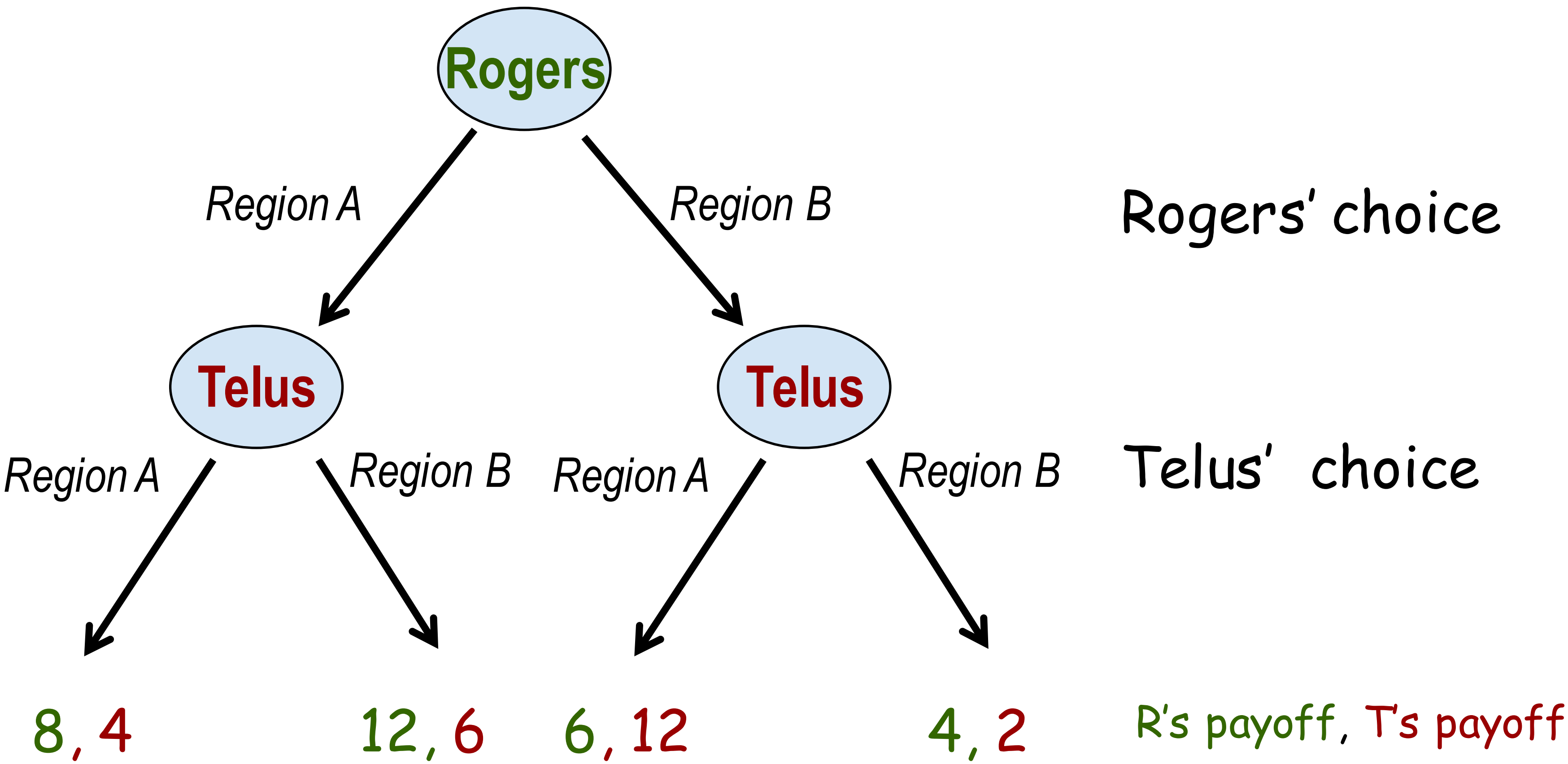
The lecture is based in part on these slides from CSC200 and the material in Chapter in Chapter 6.

Extensive Form (Dynamic) Games

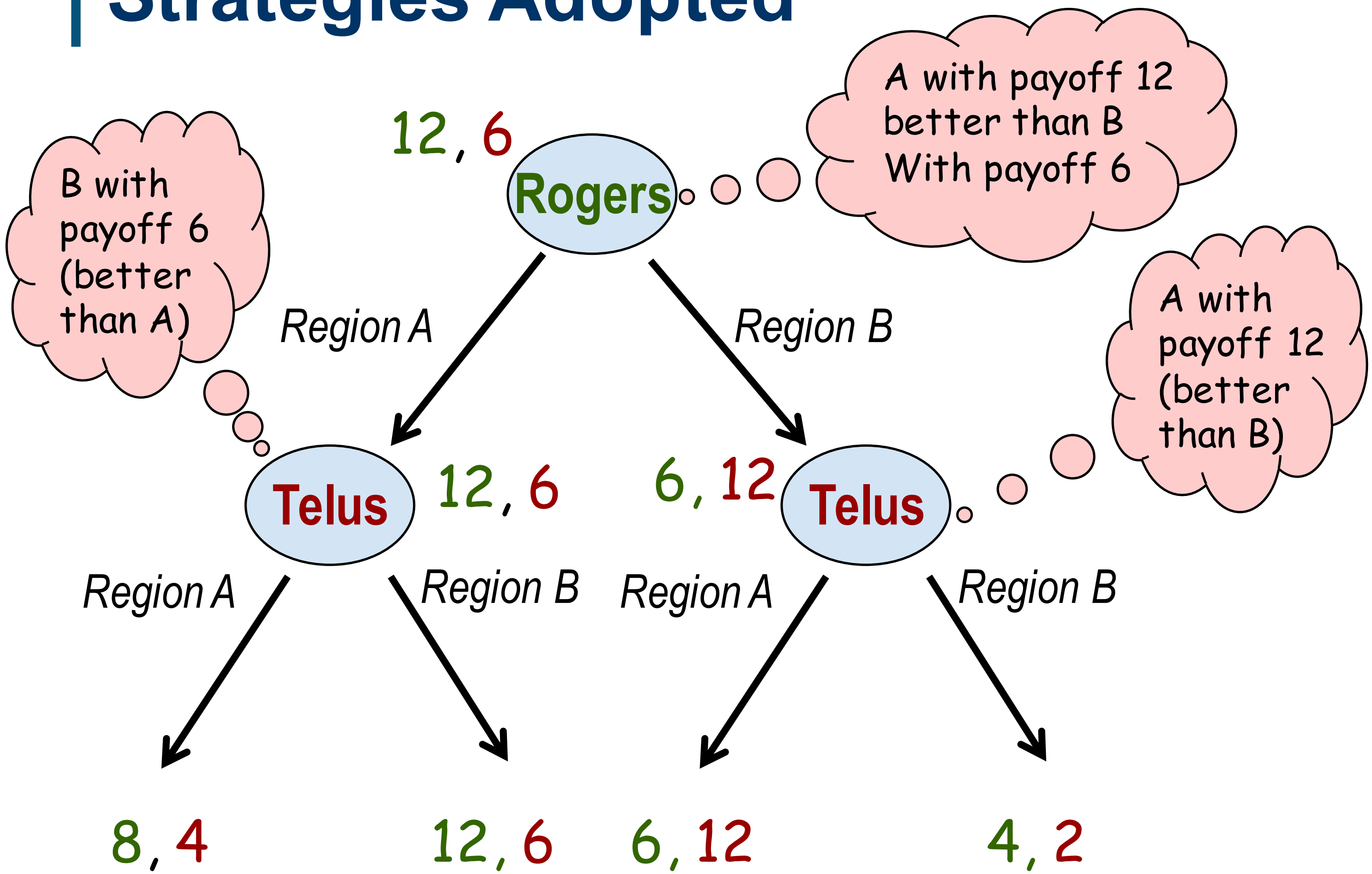
- Normal form (matrix) games seem limited
  - Players move simultaneously and outcome determined at once
  - No observation, reaction, etc.
- Most games have a *dynamic* structure (turn taking)
  - Chess, tic-tac-toe, cards games, soccer, corporate decisions, markets…
  - See what "opponent" does before making move
  - Sometimes you only see partial information (won't discuss this)

- Example: Rogers, Telus competing for market in two remote areas
  - Each firm, Rogers and Telus, can tackle one area only
  - Total revenue in Area A: 12, Area B: 6
  - If firm is alone in one area, get all of that area's revenue
  - If both firms target same area, "first mover" gets 2/3, second 1/3
  - Rogers prepped: makes first move, Telus chooses *after* Rogers
  - Critical: Telus *observes* Rogers' choice before moving!

Game Tree (Extensive Form)



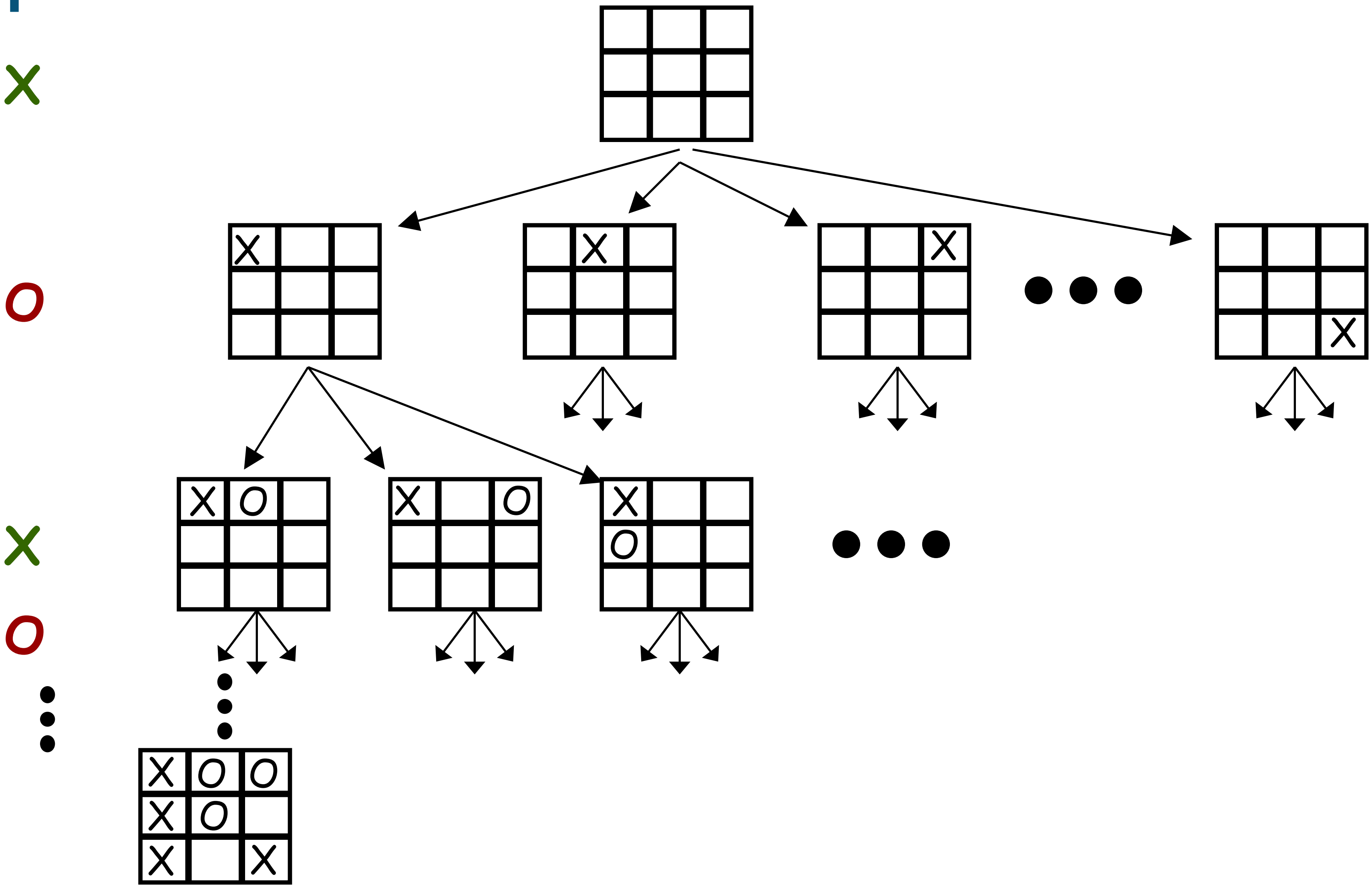
Strategies Adopted



Backward Induction

- Start from choice nodes at bottom of tree
  - Player at that node chooses best move
  - E.g., Telus chooses B at node "Rogers did A", A at node "Rogers did B"
  - This dictates which terminal node (payoffs) will be reached
  - We can now assign payoffs to that node (from chosen terminal node)
- Work up the tree: compute choices at other nodes once choices/payoffs at all child nodes made
  - Player at that node chooses best move
  - E.g., Rogers chooses A at root node
  - Dictates which child ("Rogers did A") will be reached, hence which payoff
- At end of procedure:
  - Each choice node labelled with action/choice and payoff vector
  - Payoff for the game is the payoff vector at the root node
  - Path through tree given by choices, tells us how the game will unfold

Tic-Tac-Toe Game Tree



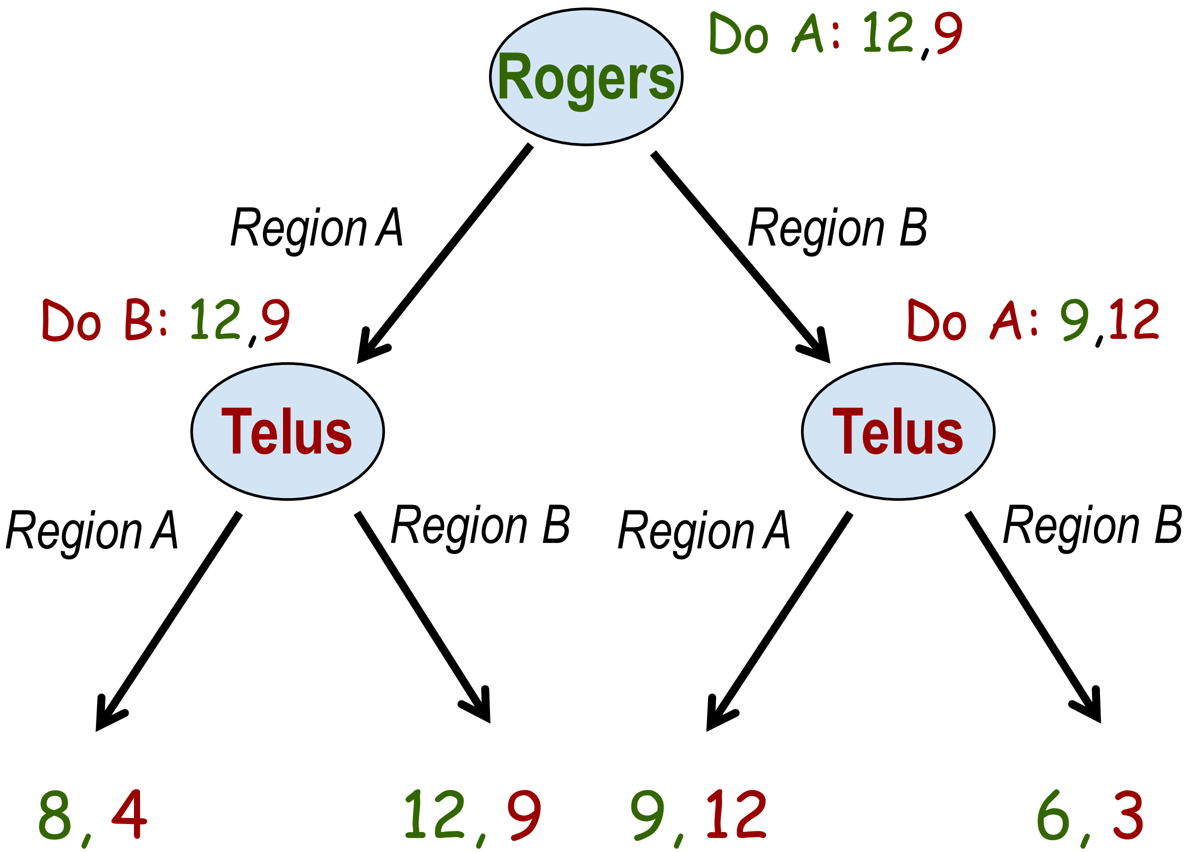
Payoff: X=1, O=-1

Conversion to Normal Form

- Extensive form game can be converted to normal (matrix) form by extending the space of strategies
- Notice Telus has *four* strategies, not just *two*
  - Two “independent” choices of A or B: “if R did A”, and “if R did B”
    - *AA,AB: A if R does A, A if R does B (i.e., do A no matter what)*
    - *AA, BB: A if R does A, B if R does B*
    - *BA, AB: B if R does A, B if R does B*
    - *BA, BB: B if R does A, B if R does B (i.e., do B no matter what)*
- Matrix game has two “identical” NE: *A/BA,AB* and *A/BA,BB*
  - Difference: what Telus does at node that won’t be reached
  - Same outcome as backward induction

		Telus			
		AA, AB	AA, BB	BA, AB	BA, BB
Rogers	A	$8, 4$	$8, 4$	$12, 6$	$12, 6$
	B	$6, 12$	$4, 2$	$6, 12$	$4, 2$

Conversion Hides Information



- Change payoffs so that region B is more valuable (9 instead of 6)
- Backward induction gives same choice of strategies (Telus gets higher payoff for region B, but otherwise the same)

Conversion Hides Information

- Notice what happens in normal form
  - The two Nash eq. ***A/BA,AB*** and ***A/BA,BB*** remain in place
  - But a new one emerges: ***B/AA,AB***
- New Nash equilibrium
  - Telus threatens to move into A if Rogers does
  - This is enough to make Rogers move to B (alone): full amount (9) of smaller payoff is better than 2/3-share (8) of larger payoff
- Why doesn't this arise in tree (backward induction)?
  - The threat is not credible (Telus sacrifices its own payoff)

Telus

Rogers	AA, AB	AA, BB	BA, AB	BA, BB
A	8, 4	8, 4	12, **9**	12, **9**
B	**9**, 12	**6, 3**	**9**, 12	**6, 3**

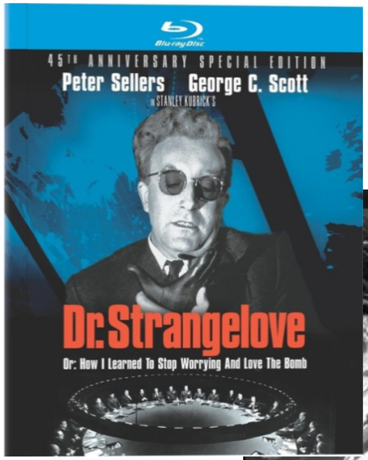
Credible vs. Non-credible Threats

- Normal form supports equilibria where second player can threaten to make a move that hurts their own payoff in order to prevent first player from taking an earlier move
  - Threat is enough to prevent the move in normal NE
  - Hence the self-inflicted damage to player will not occur

- What if first player takes the move anyway, would second player do what it threatened?
  - *The threat is not credible*, not supported by backward induction
  - We call equilibria in the game tree: *subgame perfect equilibria*
  - SPE must be Nash eq in normal form, but not all NE are SPE!

What about Pre-commitment?

- If player *could* pre-commit, the "threatening NE" is valid
- How do you make an irrevocable (or costly) commitment?
  - sign a contract
  - set loose a computer program
  - doomsday machine (Dr. Strangelove)
  - Important: first player must *know about* pre-commitment



- But this is not the same game
  - the space of strategies (precommitment moves), and the ordering of action are different than in the original game
- So subgame perfect equilibria are the most natural form of NE for dynamic (extensive form) games

The Ambassador reveals that his side has installed a doomsday device that will automatically destroy life on Earth if there is a nuclear attack against the Soviet Union. The American President expresses amazement that anyone would build such a device. But Dr. Strangelove... admits that it would be "an effective deterrent... credible and convincing.“

Strangelove explains the technology behind the Doomsday Machine and why it is essential that not only should it destroy the world in the event of a nuclear attack but also be fully automated and incapable of being deactivated. He further points out that the "whole point of the Doomsday Machine is lost if you keep it a secret".

When asked why the Soviets did not publicize this, Ambassador de Sadeski sheepishly answers it was supposed to be announced the following Monday at the (Communist) Party Congress because "the Premier loves surprises."

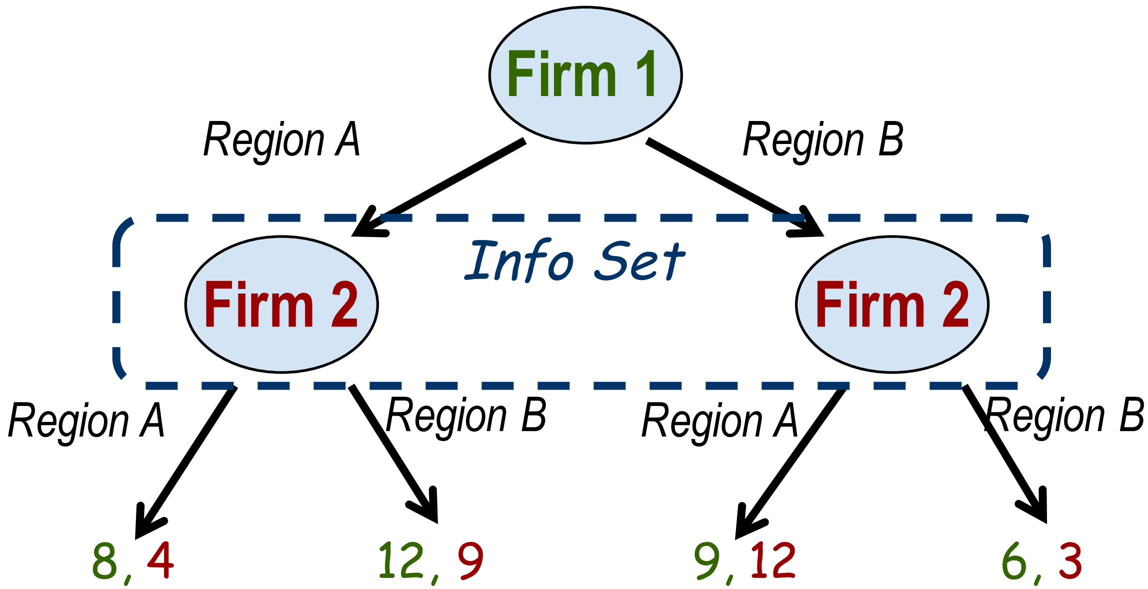
From en.wikipedia.org/wiki/Dr._Strangelove#Plot

Exercise: Voting Game

- Take our three politicians who want a raise
  - all want a raise, all want to vote no; but want raise more!
  - 2 out of 3 votes needed to pass
  - now make the game dynamic: politician 1 votes first, politician 2 votes second, politician 3 votes thirds
- Do the following
  - draw the game tree
  - identify the strategies available to each player (how many does each have?)
  - construct the normal form representation
  - what is/are its subgame perfect equilibria?
  - what is/are Nash equilibria in normal form?
  - what threats could politician 1, 2 or 3 make?

Simultaneous Moves in Extensive Form (Optional)

- Games so far are called *perfect information* games
  - Firm 2 knows move taken by Firm 1 before it makes its choice
  - Each player observes all previous moves: knows the node it's at
- Our normal form "simultaneous moves" can be represented in a tree using *information sets*
  - group together nodes (for a given player) that it cannot distinguish
  - these are called *imperfect information* games



- If 2 cannot see move made by 1, each of its two nodes is "indistinguishable"
- So must choose same action at both nodes
- Strategies select actions for each information set!