Voting and preference aggregation

CSC304 Lecture 20
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Allan Borodin
(adapted from Craig Boutilier slides)

Announcements and todays agenda

- Today: Voting and preference aggregation
  - Reading for next few classes: Ch 13(perhaps plus some ideas not discussed in the KP text)

- Announcements
  - Assignment 3 is complete with the current 5 questions. It is due November 30 at noon.
  - Comment on last lecture: When the matchings from FPDA and MPDA are different and we run the male-pessimal FPDA, then it is computationally easy to determine a man who can lie to improve his match and also easy to determine how he should do this (assuming he knows the outcome of the MPDA).
  - Student evaluation period for Arts & Science courses will open on November 25th and close on December 8th at midnight.

Social choice and voting

- A generous definition of *social choice* can include almost any topic where individual preferences (and not utilities) are the inputs.
- Social choice usually refers to the aggregation of individual preferences so as to determine a ``consensus outcome''.
- What is *voting*? Voting clearly falls under the framework of social choice and if interpreted liberally, can be an alternative meaning of social choice. The most standard sense of voting is to determine a single winner. For example, in Canada, we elect one member of Parliament for each riding.

Voting beyond a single winner

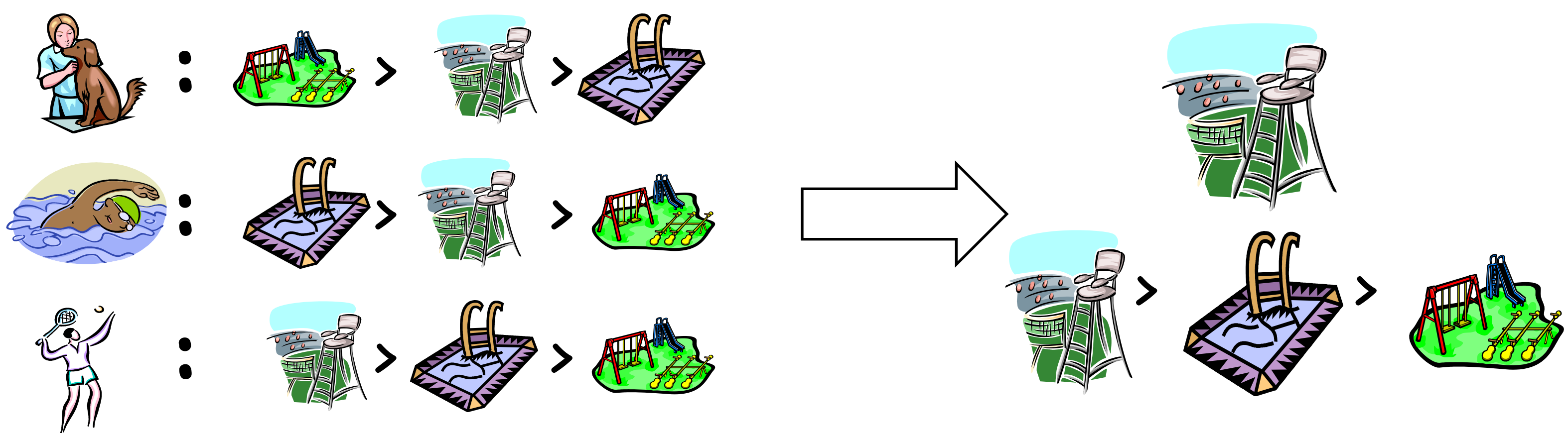
- But we sometimes do vote for more than a single winner. In some countries our votes determine the fraction of candidates from a political party to be in the legislature.

- We sometimes vote to elect a set of winners, say when a program committee selects a set of papers for a conference. Similarly, a municipality may have voting to elect a small set of ``city councillors''. In Toronto we used to vote for two ``aldermen'' for each ward.
- We could even be voting to determine a complete consensus ranking.

Voting: A Simple Example

- City has budget to build one new recreational facility: three options

- Three legislators differ in preferences over the options

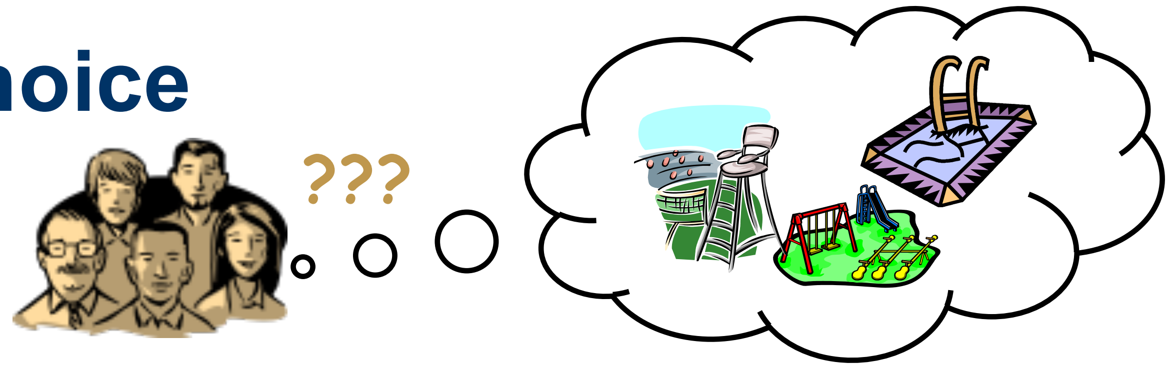


- How do we decide when we have to:
  - choose a single consensus alternative?
  - rank all three alternatives?

Voting and Preference Aggregation

- Many examples of single decisions for a group/population
  - group of friends deciding on a club, restaurant, vacation, …
  - group of businesses (or in the era of Groupon, consumers) choosing a supplier for a specific item to generate a volume discount
  - city deciding on location of new park, new bus routes, etc…
  - hiring committee selecting a job candidate
  - company designing a new product for a target market
  - search engine returning (non-personalized) search results for query *q*
  - recommender system: (non-personalized) ordering of movies, music, …
  - government setting economic, social, environmental policy
  - … **of course**, electing political representatives to some legislative body
- What's so difficult about this?
  - People have different preferences (don't agree on the best choice)
  - Need some notion of *compromise, consensus* or *group-satisfaction* to select an alternative

Social Choice

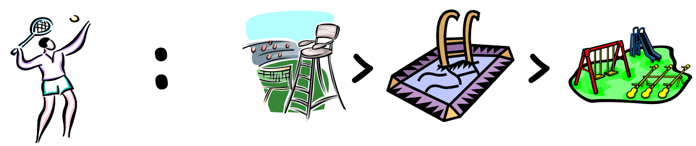


- **Social choice:** study of collective decision making
- *Aggregation of individual preferences determines a consensus outcome for some population*
    - Political representatives, committees, public projects,…
    - Studied for millennia, formally for centuries
    - Increasing importance for low stakes domains
- Key points:
    - we aggregate *preferences*, not judgments/opinions (for now)

    - preferences are qualitative: *rankings*, not utilities or valuations
        - looks like mechanism design (e.g., for designing auctions) but without valuations and monetary transfers
        - can be difficult to compare, add, average preferences

Individual Preferences

- Assume a finite set of alternatives *A* (e.g., rec facilities)
- A person's preferences is a *total linear ordering (ranking)* of *A*
  - Picture is the same as when we discussed Gale-Shapley matching



- Ordering is equivalent to requiring that a person's preference be:
  - **complete:** everything comparable; either $a \succ b$ or $b \succ a$ for any a,b in A
  - **transitive:** if $a \succ b$ and $b \succ c$, then $a \succ c$
- Completeness usually important (though allowing ties is reasonable)
  - otherwise when faced with two choices $\{a,b\}$, person is unable to decide
- Transitivity important to prevent cyclic (strict) preferences
  - violates certain rationality principles (e.g., the "money pump")

Voting Systems

- Assume:
  - m alternatives or candidates $A = \{a_1, \ldots, a_m\}$
  - n individuals or voters $N = \{1, \ldots, n\}$ with preferences over A

- A *voting system* or *rule* accepts the preferences of N as input and aggregates them to determine either:
  - a *winner* or consensus alternative from A
  - a *group/consensus ranking (or top k ranking)* of the alternatives
  - Note: approval voting doesn't quite fit this definition

- This is a broad definition! How do we go about choosing a reasonable voting rule?
  - Let's focus on picking winners for now (not rankings)
  - Let's start by looking at a few examples

Plurality Voting

- *Plurality voting:*
  - **Input:** rankings of each voter
  - **Winner:** alternative ranked 1st by greatest number of voters
    - number of 1st-place rankings is *a*'s *plurality score*
    - *complete* rankings not needed, just votes for most preferred alternatives
    - we'll ignore ties for simplicity
  - This is a most familiar scheme, used widely:
    - locally, provincially, nationally for electing political representatives
  - With only 2 alternatives, often called *majority voting*

- Example preference profile (three alternatives):
  - $A \succ B \succ C$: 5 voters
  - $C \succ B \succ A$: 4 voters
  - $B \succ C \succ A$: 2 voters
- Winner: A wins (plurality scores are A: 5; C: 4; B:2)

The Borda Rule

- *Borda voting rule:*
  - **Input:** rankings of each voter
  - *Borda score* for each alternative *a*: *a* gets m-1 points for every 1st-place rank, *m-2* points for every 2nd-place, etc.
  - **Winner:** alternative with highest Borda score
  - Used in sports (Heismann, MLB awards), variety of other places
  - Proposed by Jean-Charles, chevalier de Borda in 1770 to elect members to the French Academy of Sciences (also Ramon Llull, 13th century)

- Example profile (three alternatives, positional scores of 2, 1, 0):
  - $A \succ B \succ C$: 5 voters
  - $C \succ B \succ A$: 4 voters
  - $B \succ C \succ A$: 2 voters
- Winner: B wins (Borda scores are: B: 13; A: 10; C: 10)
  - Notice: more sensitive to *the entire range of preferences* than plurality (which ranked *B* last)

Approval Voting

- Approval Voting
  - **Input:** voters specify a *subset* of alternatives they "approve of"
  - Approval score: a point given to a for each approval
    - variant: *k*-approval, voter lists exactly *k* candidates
  - **Winner:** alternative with highest approval score
  - used in many informal settings (at UN, Doge of Venice, …)
  - Steven Brams a major advocate (see Wikipedia article)

- Example profile (three alternatives, approvals in bold):
  - **A** $\succ$ B $\succ$ C: 5 voters (approve of only top alternative)
  - **C** $\succ$ B $\succ$ A: 4 voters (approve of only top alternative)
  - **B** $\succ$ **C** $\succ$ A: 2 voters (approve of top two alternatives)
- Winner: C wins (approval scores are: C: 6; A: 5; B: 2)
  - Notice: can't predict vote based on ranking alone!

Positional Scoring (Voting) Rules

- Observe that plurality, Borda, *k*-approval, *k*-veto are all each *positional scoring rules*
- Each assigns a *score* $\alpha(j)$ to each rank position j
  - almost always non-increasing in j
- The winner is the candidate a with max total score: $\sum_i \alpha(r_i(a))$

In general:	$\alpha(1)$	$\alpha(2)$	$\alpha(3)$	$\alpha(4)$
Plurality:	1	0	0	0
Borda:	3	2	1	0
2-Approval:	1	1	0	0
1-Veto:	1	1	1	0
Could be :	10	2	0	0

Which of these is Better?

- Notice that on the same vote profile, plurality, Borda, and approval gave different winners!

- Which is best?
  - hard to say: depends on social objective one is trying to meet
  - common approach: identify *axioms/desirable properties* and try to show certain voting rules satisfy them
    - we will see it is not possible in general to satisfy all axioms!

- But let’s look at a few more voting rules just to get a better sense of things.

There are Hundreds of Voting Rules

- *Single-transferable vote (STV)* or *Hare* system
  - Round *1*: vote for favorite candidate; eliminate candidate with lowest plurality score;
  - Round *t*: if your favorite is eliminated at round *t-1*, recast vote for favorite remaining candidate; eliminate candidate with lowest plurality score
  - Round *m-1*: winner is last remaining candidate if not chosen sooner
    - terminate at any round if plurality score of top candidate is at least *n/2* (i.e., there is a majority winner)
  - Used: Australia, New Zealand, Ireland, some political party conventions

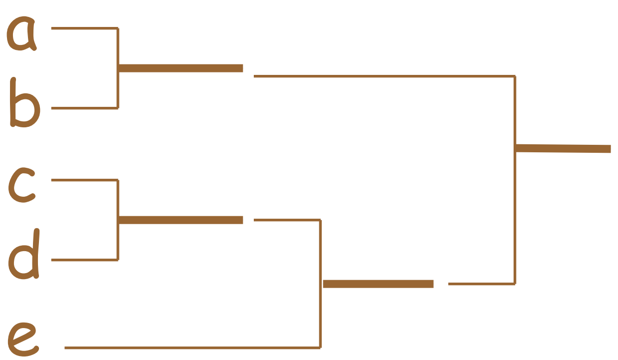
    Doesn't necessitate repeated voting: voters can submit rankings once
  - When would this be a bad voting rule?
- *Nanson's rule*
  - Just like STV, but use Borda score to eliminate candidates

There are Hundreds of Voting Rules

- *Egalitarian (maxmin fairness)*
  - Winner maximizes minimum voter's rank: $argmax_a \, min_j \, (m - r_j(a))$

- *Copeland*
  - Let $W(a,b,\mathbf{r}) = 1$ if more voters rank $a \succ b$; 0 if more $b \succ a$; ½ if tied
  - Score $s_c(a,\mathbf{r}) = \sum_b W(a,b,r)$; winner is a with max score
    - *i.e., winner is candidate that wins most pairwise elections*

- *Tournament/Cup*
  - Arrange a (usually balanced) tournament tree of pairwise contests
  - Winner is last surviving candidate
  - We'll discuss this in more detail later



Condorcet Principle

- How would you determine "societal preference" between a pair of alternatives *a* and *b*?
- A natural approach: run a "pairwise" majority vote: if a *majority* of voters prefer *a* to *b*, then we say *the group prefers a to b*
- *Condorcet winner:* an alternative that beats every other in a pairwise majority vote
  - proposed by Marie Jean Antoine Nicolas de Caritat, marquis de Condorcet in 1785
  - if there is a Condorcet winner, it must be unique
  - a rule is *Condorcet-consistent* if it selects the Condorcet winner (if one exists)

- Condorcet winners need not exist (next slide)
  - and many natural voting rules are not Condorcet consistent (e.g., plurality, Borda, STV are not), but some are: Nanson, Copeland, Cup, etc.

Condorcet Paradox

- *Condorcet paradox:*
  - suppose we use the pairwise majority criterion to produce a societal preference ranking
  - pairwise majority preferences may induce *cycles* in societal ranking (i.e., the preference ranking is not transitive)
- Simple example:
  - $A \succ B \succ C$: *m/3* voters
  - $C \succ A \succ B$: *m/3* voters
  - $B \succ C \succ A$: *m/3* voters
  - Societal ranking has $A \succ B$, $B \succ C$, and $C \succ A$ (!)
  - No clear way to produce a consensus ranking
  - Also evident that this preference profile has no Condorcet winner

Violations of Condorcet Principle

- Plurality violates Condorcet
  - 499 votes: $A \succ B \succ C$
  - 3 votes: $B \succ C \succ A$
  - 498 votes: $C \succ B \succ A$
  - plurality choses A; but B is a CW (B>A 501:499; B>C 502:498)

- Borda violates Condorcet
  - 3 votes: $A \succ B \succ C$
  - 2 votes: $B \succ C \succ A$
  - 1 vote: $B \succ A \succ C$
  - 1 vote: $C \succ A \succ B$
  - Borda choses B (9 pts) ; but A is a CW (A>B 4:3; A>C 4:3)
  - notice *any positional* scoring rule (not just Borda) will choose B if scores strictly decrease with rank

The Axiomatic Method

- Considerable work studies various *“axioms”* or principles that we might like voting rules to satisfy and ask whether we can devise rules that meet these criteria. This seems like a ``principled’’ way to proceed rather than just try to compare various voting rules without knowing what we properties we really (most) want.

- For example, the Condorcet principle is an axiom/property we might consider desirable. We’ve seen some voting rules satisfy it, and others do not.

If time permits we will consider a few more rather intuitive axioms.