Homework Assignment 3

CSC 304F: Algorithmic game theory and mechanism design

Due: Noon, Wednesday, November 30, 2016. Note that we have extended the due date to give more time for topics we are now discussing.

Be sure to include your name and student number with your assignment. If your handwriting is *possibly* illegible, be sure to hand in your assignment in some typed form. All assignments must be submitted using the online Markus submission. Preferably, you can use latex or powerpoint and then convert to pdf. If handwritten, then make pdf from a copier.

Please take note of the comments regarding assignments that are given in the course information sheet. In particular, take note of the "20%" rule.

1. Consider the following combinatorial auction with 2 agents and 2 objects $\{a, b\}$. The valuation of agent 1 is $v_1(\{a\} = 14, v_1(\{b\}) = 1$ and the valuation of agent 2 is $v_2(\{a\} = 10, v_2(\{b\}) = 10, v_2(\{a, b\}) = 30$.

   - What is the allocation and payments for the VCG mechanism.
   - Consider the greedy algorithm that sorts all the 5 individual bids $\{b_i(S_i)\}$ by non-increasing value of $r(i, S) = \frac{b_i}{|S|}$. For this CA, if truthful agent 2's bid for $S = \{a, b\}$ would come first and then agent 1's bid for $S = \{a\}$. The algorithm considers each bid in order and accepts the bid (i.e. allocates the requested set) if that set does not intersect with a previously allocated set and the agent has not already been allocated a set. Suppose that we have a mechanism that uses the above greedy algorithm and for each accepted bid (b_i, S_i) charges $|S_i| \cdot \frac{b_j}{|S_j|}$ where agent j is the next agent to be allocated a non-empty set S_j. (If S_i is the last set allocated, then agents i pays nothing. That is, each agent is being charged per item at the next lowest price per item.

     What is the allocation and prices for this greedy mechanism.
   - Is this mechanism truthful? If yes, indicate how truthfulness follows from some known result. If not, which agent could bid non truthfully so as to result in higher utility (say assuming the other agent bids truthfully).

2. In the lecture on Wednesday, November 9, I stated that using a potential argument, the ascending algorithm must terminate and will result in an allocation that is optimal for social welfare. Prove these statements. Hint: This is easy but not as easy as I first thought. I will explain in class (Monday, November 21) the one somewhat subtle point.

3. Consider a matching market in which there are four condos, C1, C2, C3, and C4, for sale in a certain neighborhood. There are four buyers Dan, Etta, Fahiem, and Gabor wishing to purchase these condos. The buyers have the following valuations:

	C1	C2	C3	C4
Dan	2	7	6	4
Etta	7	9	5	4
Fahiem	3	2	5	4
Gabor	4	7	2	1

(a) Suppose we run the "auction" mechanism described in lectures 16 and 17 to determine market clearing prices, but *without using the price reduction step (i.e., we do not insist that the smallest price is* 0*).* as discussed in lecture 19. Recall that at each round of the auction, if there is a constricted set of buyers in the preferred seller graph, then we trigger price increases for one or more condos. We want you to run this auction using the following rule: *if there is more than one possible constricted set of buyers at any round of the auction, choose any* ***minimal*** *set of constricted buyers to determine which condo prices increase.* For each round of the auction, show each of the following: i. prices for each condo; ii. the preferred seller graph; iii. if the prices are not market clearing at that round, the *minimal* constricted set of buyers you selected and the condos whose prices will increase; and iv. if the prices are market clearing at that round, a perfect matching that assigns a condo to each buyer.

(b) Repeat part (a), but this time, if there is more than one possible constricted set of buyers at any round of the auction, choose a **maximal** set of constricted buyers to determine which condo prices increase. Be sure to show all details of every round as in part (a). How do the market clearing prices reached compare to those in part (a)?

(c) Starting from the market clearing prices computed in part (a), suppose the sellers each decided to increase their prices by the same amount i. What is the largest price increase i they could impose without changing the preferred seller graph?

(d) Suppose another very attractive condo C5 comes onto the market. All four buyers value this condo more than any of the other four condos. The owner of this condo recognizing its attractiveness prices it higher than any of the buyer valuations. (i) Briefly explain why the addition of this condo (at this price) won't change the market clearing prices of the other condos. (ii) In real-world real estate markets, the entrance of a new condo into the market at a high price will often cause prices for other condos in the neighborhood to increase in value. Explain what feature(s) of the market clearing model we are consideruing prevents this from happening, and what feature(s) of real-world real estate might cause this to occur.

4. Consider a stable marriage problem with four women Xina, Yael, Zoe and Winnie, and four men, Arnold, Bikash, Cam, and Denzel. Using initials instead of names, the women's preference for the men are:

$$X : A \succ B \succ C \succ D \qquad Y : A \succ B \succ C \succ D \qquad Z : B \succ D \succ A \succ C \qquad W : C \succ A \succ B \succ D$$

And the men's preferences for the women are:

$$A : Z \succ W \succ X \succ Y \qquad B : X \succ Y \succ Z \succ W \qquad C : X \succ Y \succ W \succ Z \qquad D : Y \succ W \succ X \succ Z$$

(a) Suppose we run the female-proposing deferred acceptance (FPDA) algorithm. Show how each iteration will proceed by: (i) clearly labelling each iteration; (ii) stating exactly which proposals will be made at that iteration; (iii) and stating exactly which engagements will be in place at the end of that iteration (once relevant proposals are accepted or rejected). Indicate clearly which iteration is the final one and what stable marriages result from FPDA.

(b) We say a man *lies* in FPDA when he rejects a proposal from a woman even though he prefers the proposer to his current fiancee. Identify some man who can *lie* by falsely rejecting a proposal, thereby changing the outcome of FPDA so that he ends up married to a more preferred partner that he did in part (a). State when he should lie in FPDA and what stable marriages will result. (Assume all other men continue to accept and reject proposals truthfully.)

(c) Now consider the *male-proposing* deferred acceptance (MPDA) algorithm on the same problem (i.e., same set of preferences). As in part (a), clearly indicate what happens at each iteration of MPDA and what stable marriages result when MPDA terminates.

5. Suppose we run a plurality election to determine a winner from a set of four candidates a, b, c, d. Assume that any ties are broken in alphabetical order (e.g., if a and c tie with the most votes, then a is declared the winner). Consider the following profile of eight voter preferences. For convenience, the plurality scores for each candidate are also shown (but you're encouraged to verify them):

Voter	V1	V2	V3	V4	V5	V6	V7	V8
more preferred	c	c	b	b	a	a	c	c
↓	b	b	a	c	b	d	b	b
	d	a	d	d	c	b	a	d
less preferred	a	d	c	a	d	c	d	a

	Plurality Score	Borda Score
a	2	10
b	2	17
c	4	15
d	0	6

- Assuming everyone votes sincerely. Can any single voter change her vote (assuming everyone else votes sincerely) and change the result of the plurality election so that she prefers the new winner to the original winner c? If so, briefly explain how (who should change her vote, how she would change it, and who the new winner would be). If not, briefly explain why not.
- Now suppose we use the Borda rule for this election. The Borda scores, assuming sincere voting, are shown above. (Again, you are encouraged to verify the scores.). The winner would now be b rather than c. The four voters $(V1, V2, V7, V8)$ whose favorite candidate is c do not know how the others will vote, but strongly suspect that b is very popular. With this suspicion, how do you think they should report, or *misreport*, their preferences to increase the odds of their favorite candidate c winning? If they all vote this way, will c win? (Assume the other four voters vote sincerely.)
- The government suddenly discovers that there was an additional voter V9 whose ranking (and vote) was not counted. The preference ranking for V9 is $b \succ c \succ a \succ d$. In fairness, the government decides to include V9s preferences and now decides to use the single transferable vote (STV) rule for voting (using plurality to drop off the candidate with the lowest score and breaking ties in favour of the candidate who wins the most pairwise comparisons). Assuming all nine voters will vote sincerely, determine the winner of this election. Describe each round in this election.