

Homework Assignment 2

CSC 304F: Algorithmic game theory and mechanism design

Due: October 28, 2016

Be sure to include your name and student number with your assignment. If your handwriting is possibly illegible, be sure to hand in your assignment in some typed form. All assignments must be submitted using the online Markus submission. Preferably, you can use latex or powerpoint and then convert to pdf. If handwritten, then make pdf from a copier.

Please take note of the comments regarding assignments that are given in the course information sheet. In particular, take note of the “20%” rule.

1. QuickRecognition (QR) is a new “crowdsourcing” service for *photo recognition*: it allows individuals to submit photos and returns an appropriate description of the scene. Photo recognition requires two substeps: (a) identifying objects in the photo, and (b) generating a textual description of the photo based on the objects. QR employs two types of workers: *photo experts* who can quickly identify objects, but are slow at generating the required text; and *text expert*, who are slow at identifying objects, but can quickly generate the required text.

Due to an old management-union agreement, QR can currently only assign photos to one worker, either a photo specialist or a text specialist. It has no ability to split the task over two different specialists. When an individual submits a photo to QR, it specifies whether it wants the photo to be labeled by a photo expert or a text expert. There is no difference in the cost or quality of the result, but the time taken depends on how busy the photo experts and text experts are on a given day. Specifically,

- If the photo experts are given x jobs on a given day, it will take them $\frac{x}{20}$ minutes to identify objects and 55 minutes to produce textual descriptions (no matter how many jobs there are). So the total time taken to complete x jobs is $\frac{x}{20} + 55$ minutes.
- If the textual specialists are given x jobs on a given day, it will take them 65 minutes to identify the objects (no matter how many jobs there are), and $\frac{x}{20}$ minutes to produce textual descriptions. So the total time taken to complete x jobs is $65 + \frac{x}{20}$ minutes.

Hint: you can “visualize” this problem something like a traffic network, as in Ch.8 of the KP text. Each photo must get from a starting point (uncaptioned) to a destination (captioned). There are two “routes,” corresponding to whether the photo is processed by a photo expert or a text expert, and the time taken depends on how many photos use each “route”:

There are 1000 individuals that use QR each day, and each submits one photo per day (so QR processes 1000 photos per day). All 1000 individuals submit their jobs simultaneously, stating which process they want to use: PE (photo expert) or TE (text expert). Every individual wants their results as quickly as possible.

- (a) In equilibrium, how many individuals will request PE and how many will request TE? What will be the total processing time faced by an individual in each category? Justify your answer by demonstrating that it is indeed in equilibrium.
- (b) Given a new management-union agreement, QR has developed a new procedure that allows the task of labeling a photo to be split up among the two types of specialists: a photo can now be processed by a photo expert who identifies the objects in the photo, and then passes it on to a text expert, who generates a caption based on the objects identified by the photo expert. Individuals submitting photos can now specify whether they want to use the original PE process, the original TE process, or this new combined process (we'll call it PT). The time for processing photos using the PT process is the sum of the times needed to do the object identification and the text generation by the appropriate specialists. These are affected by how many photos use the old processes. Specifically, with x photos processed using PE, y photos using TE, and z photos using PT, we have:
- Time for PE photos is $\frac{x+z}{20} + 55$.
 - Time for TE photos is $65 + \frac{y+z}{20}$.
 - Time for PT photos is $\frac{x+z}{20} + \frac{y+z}{20}$.

We still have 1000 individuals submitting photos each day. With this new procedure in place, in equilibrium, how many individuals will request PE, how many TE, and how many PT? What happens to the processing time faced by individuals relative to part (a)? Justify your answer by demonstrating that it is indeed in equilibrium.

- (c) QR realizes that the new process has made their clients a little worse off because of your answer in part (b). So they decide to introduce an artificial delay of t minutes to the combined process: the effect of this is to increase the completion time of PT by t minutes. Hence, the formula in part (b) for PT processing time changes to:

$$\frac{x+z}{20} + \frac{y+z}{20} + t$$

You can view part (b) as having a delay of 0 minutes. Adding a delay of 1 minute should not change equilibrium flow. Adding a delay of 120 minutes should clearly change the equilibrium flow (indeed, you would expect that such a large delay would push *everyone* away from the combined process). By answering the following question, you will help FastCrowd understand the impact of these delays:

Is there any delay that QR can impose that will lead to a better equilibrium or would it be possibly better to charge more for the PT procedure? Explain your answer.

2. (Exercise 14.2.5 in KP) Show that the sealed bid k unit Vickery auction is truthful; That is, the auction allocates to the k highest bidders at a price equal to the $(k+1)^{st}$ highest bid.
3. Suppose there are two i.i.d. buyers with identical distributions $U[0, 2]$.
 - (a) Determine the bids and expected revenue for a 1st price auction.
 - (b) Determine the expected revenue using the Myerson optimal auction.