Assignment 2: sample Solutions

CSC 304F: Algorithmic game theory and mechanism design

Due: October 28, 2016

Be sure to include your name and student number with your assignment. If your handwriting is *possibly* illegible, be sure to hand in your assignment in some typed form. All assignments must be submitted using the online Markus submission. Preferably, you can use latex or powerpoint and then convert to pdf. If handwritten, then make pdf from a copier.

Please take note of the comments regarding assignments that are given in the course information sheet. In particular, take note of the "20%" rule.

1. QuickRecognition (QR) is a new "crowdsourcing" service for *photo recognition*: it allows inddiviviudals to submit photos and returns an appropriate description of the scene . Photo recognition requires two substeps: (a) identifying objects in the photo, and (b) generating a textual description of the photo based on the objects. QR employs two types of workers: *photo experts* who can quickly identify objects, but are slow at generating the required text; and *text expert*, who are slow at identifying objects, but can quickly generate the required text.

   Due to an old management-union agreement, QR can currently only assign photos to one worker, either an photo specialist or a text specialist. It has no ability to split the task over two different specialists. When an individual submits a photo to QR, it specifies whether it wants the photo to be labeled by an photo expert or a text expert. There is no difference in the cost or quality of the result, but the time taken depends on how busy the photo experts and text experts are on a given day. Specifically,

   - If the photo experts are given x jobs on a given day, it will take them $\frac{x}{20}$ minutes to identify objects and 55 minutes to produce textual descriptions (no matter how many jobs there are). So the total time taken to complete x jobs is $\frac{x}{20} + 55$ minutes.
   - If the textual specialists are given x jobs on a given day, it will take them 65 minutes to identify the objects (no matter how many jobs there are), and $\frac{x}{20}$ minutes to produce textual descriptions. So the total time taken to complete x jobs is $65 + \frac{x}{20}$ minutes.

   Hint: you can "visualize" this problem something like a traffic network, as in Ch.8 of the KP text. Each photo must get from a starting point (uncaptioned) to a destination (captioned). There are two "routes," corresponding to whether the photo is processed by a photo expert or a text expert, and the time taken depends on how many photos use each "route":

   There are 1000 individuals that use QR each day, and each submits one photo per day (so QR processes 1000 photos per day). All 1000 individuals submit their jobs simultaneously, stating which process they want to use: PE (photo expert) or TE (text expert). Every individual wants their results as quickly as possible.

(a) In equilibrium, how many individuals will request PE and how many will request TE? What will be the total processing time faced by an individual in each category? Justify your answer by demonstrating that it is indeed in equilibrium.

Answer: In equilibrium, 600 individuals will request will request PE and 400 will request TE.
To verify: Each process will take $600/20 + 55 = 65 + 400/20 = 85 minutes$
To derive: $x/20 + 55 = 65 + (1000 - x)$
Any inidivual who switches to a new process, will cause that proceess (and themselves to finish later.

(b) Given a new management-union agreemnt, QR has developed a new procedure that allows the task of labeling a photo to be split up among the two types of specialists: a photo can now be processed by a photo expert who identifies the objects in the photoo, and then passes it on to a text expert, who generates a caption based on the objects identified by the photo expert. Individuals submitting photos can now specify whether they want to use the original PE process, the original TE process, or this new combined process (we'll call it PT). The time for processing photos using the PT process is the sum of the times needed do the object identification and the text generation by the appropriate specialists. These are affected by how many photos use the old processes. Specifically, with x photos processed using PE, y photos using TE, and z photos using PT, we have:

- Time for PE photos is $\frac{x+z}{20} + 55$.
- Time for TE photos is $65 + \frac{y+z}{20}$.
- Time for PT photos is $\frac{x+z}{20} + \frac{y+z}{20}$.

We still have 1000 individuals submitting photos each day. With this new procedure in place, in equilibrium, how many individuals will request PE, how many TE, and how many PT? What happens to the processing time faced by individuals relative to part (a)? Justify your answer by demonstrating that it is indeed in equilibrium.

Answer: This is an example of the Braess paradox. An equilibrium will be when everyone chooses the new process PT.
In equilibirum, the time for process PT is 1000/20 + 1000/20 = 100.
The time for PE is 1000/20 + 55 = 105 and for TE is 65 + 1000/20 = 115 so that no one wouldd want to switch.

(c) QR realizes that the new process has made their clients a little worse off because of your answer in part (b). So they decide to introduce an artificial delay of t minutes to the combined process: the effect of this is to increase the completion time of PT by t minutes. Hence, the formula in part (b) for PT processing time changes to:

$$\frac{x+z}{20} + \frac{y+z}{20} + t$$

You can view part (b) as having a delay is 0 minutes. Adding a delay of 1 minute should not change equilibrium flow. Adding a delay of 120 minutes should clearly change the equilibrium flow (indeed, you would expect that such a large delay would push *everyone* away from the combined process). By answering the following question, you will help FastCrowd understand the impact of these delays:

Is there any delay that QR can impose that will lead to a better equilibirum or would it be possibly better to charge more for the PT procedure? Explain your answer.

Answer: In equilibirium, we would like all three processes to take the same time.
If we wanted to prevent anyone from using the combined PT process we should charge at least 35 since then the combined process would take 1000/20 + 35 = 85 which we know we can obtain when we didn't have the comined process. But a delay of less than 35 would still enable the possibility of inidividuals using the combined process.

If the delay is t minutes then the time for PT would be $\frac{x+z}{20} + \frac{y+z}{20} + t$ Any delay t with $0 \leq t < t$ would not allow reaching the equilibirum that we can obtain without the PT process.

$$\frac{x+z}{20} + 55 = 65 + \frac{y+z}{20}$$

Informally, we can see this by considering the continuous process of raising the delay from 0 (at which point the equilibirum is 100) to 35 at which point we reach the optimum equilibrium of 85.

While we cannot reduce the equilibirum completion time for everyone by adjusting the delay penalty t, QR can at least raise additional revenue by offering a shorter completion time. The idea would be to set a price to force a limited number of people who can afford and will want the PT process while others will not.

2. (Exercise 14.2.5 in KP) Show that the sealed bid k unit Vickery auction is truthful; That is, the auction allocates to the k highest bidders at a price equal to the $(k+1)^{st}$ highest bid.

   Answer: As I mentioned in class, you are not allowed to just say that this auction is the VCG auction and hence truthful. You were to prove the result by a direct argument similar to the proof for the single item Vickrey auction.

   We have to see why it is never in anyones benefit (in terms of utility) to not bid truthfully (no matter what the other agents bid. We can assume n bidders with values $v_1 \geq v_2 \geq v_k \ldots \geq v_n$.

   - Consider an arbitrary agent i who won (ie one of the top k bidders). By changing their bid they cannot obtain more value and only can possibly become a loser by lowering their value.
   - Consider an arbitrary agent i who lost. This would be a bidder (say agent i with the ith highest value who has bid at most b_k. If $v_i \leq b_k$, then to win he will have to bid more than b_k and hence have negative utility when "winning". If $v_i > b_k$ and i lost, then clearly he should have bid at least v_i to hope to win. That is, there was no benefit in underbidding.

3. Suppose there are two i.i.d. buyers with identical distributions $U[0, 2]$.

   (a) Determine the bids and expected revenue for a 1st price auction.

   Answer: This is very similar to the situation when the distribution is i.i.d. $U[0, 1]$. if this was a truthful second price auction. Arguing informally (although this can be made formal), $\mathbb{E}[max(v_1, v_2)] = 4/3$ and $\mathbb{E}[min(v_1, v_2)] = 2/3$. From revenue equivalence, we know that the expected revenue is the same as if this was a truthful second price auction so that the expected revenue is $\mathbb{E}[\min(v_1, v_2)] = 2/3$. As for $U[0, 1]$, for two agents with i.i.d. distibutions in $U[0, h]$, the bid for value x is still $\beta(x) = x/2$.

(b) Determine the expected revenue using the Myerson optimal auction.

Answer:

We know that the Myerson auction for i.i.d bidders with increasing virtual values for a single item is to use a Vickrey (i.e. 2nd price) auction with a reserve price of $\psi^{-1}(0)$ where ψ is the virtual value $\psi(v) = v - \frac{1-F(v)}{f(v})$. For the distribution $U[0, 2]$, $F(v) = v/2$ and $f(v) = 1/2$. Therefore, $\psi(v) = v - \frac{1-v/2}{1/2} = 2v - 2$ and then the reserve price p^* is $\psi^{-1}(0) = 1$.
Since this is a Vickrey auction and the virtual value is increasing, we can assume that bidders have bid truthfully so the expected revenue can be calculated by looking at two mutually exclusive cases:

(a) Both bids are above the critical price.
The expected revenue from this case is

$$Prob[\min(v_1, v_2) \geq 1] \cdot \mathbb{E}[\min(v_1, v_2)| \min(v_1, v_2) \geq 1] = (1/2)^2 \cdot \frac{4}{3} = \frac{1}{3}$$

.

(b) The highest bid is above but the next highest is below. Here the revenue is 1, the reserve price.

$$Prob[\min(v_1, v_2) \geq 1, \max(v_1, v_2) \geq 1] \cdot 1 = \frac{1}{2} \cdot 1 = \frac{1}{2}$$

Hence the expected revenue is $\frac{5}{6}$