Homework Assignment 1

CSC 304F: Algorithmic game theory and mechanism design

Due: October 7, 2016

Be sure to include your name and student number with your assignment. If your handwriting is *possibly* illegible, be sure to hand in your assignment in some typed form. Instructions for online submission will be provided. Using latex or powerpoint and then converting to pdf is what we prefer.

Please take note of the comments regarding assignments that are given in the course information sheet. In particular, take note of the "20%" rule.

1. Consider the following game in matrix form with two players. Payoffs for the row player Shelia are indicated first in each cell, and payoffs for the column player Thomas are second.

	C	D
A	$10, 16$	$14, 24$
B	$15, 20$	$6, 12$

   (a) Does either player have a dominant strategy? Explain your answer.

   (b) What are the *pure* strategy Nash equilibria (if any) of this game? Justify your answer. If there is more than one pure equilibria, which would Thomas prefer? What is the Price of Anarchy (with respect to pure NE) for this game?

   (c) This game has a *fully mixed* strategy Nash equilibrium in which both Shelia and Thomas play each of their actions with positive probability. What are the mixed strategies for each player in this equilibrium? Show how you would compute such a mixed equilibrium and verify that your mixed strategies are indeed in equilibrium.

   (d) Suppose that Shelia and Thomas play this game repeatedly once per day, each time choosing their actions according to some strategy. Shelia claims that she will play her mixed strategy according to the probabilities you calculated in part (c). Thomas decides to take Shelia at her word and after a few days of playing his mixed strategy (as above), he commits to play a pure strategy from now on. Does it matter which one he plays and if so which one will he play?

   (e) After Thomas commits to play a pure strategy as in part (d), should Shelia reneg on her word and play a different strategy knowing that Thomas has committed to a pure strategy and will never change? Why or why not?

2. (a) Consider an arbitrary 2 by 2 two person game in which neither player has a dominant strategy. Prove the following: In such a game, every NE is either a pure NE or a fully mixed NE.

   (b) Show that the above statement is not necessarily true for every 2 by 3 two person game; that is, exhibit a 2 by 3 game with no dominating strategies that has an NE with one player playing a pure strategy and the other playing a (non pure) mixed strategy.

3. Consider the following modification/clarification of exercise 2.12 in the KP text. Company I chooes one of the three locations (L,C,R) and simultaeously Company II places two restaurants at two different locations. While Company II will not place two restaurants in the same location, it is possible that both companies will have a restaurant in the same location.

   This is being phrased as a zero-sum game where the value of the game is the probability that a customer visits the restaurant of Company I. Without loss of generality we can assume that a customer always knows which location is closest (i.e. does not have to worry about two locations being at the same distance).

   (a) Express this as a 3 by 3 zero-sum game in matrix form.

   (b) Express the value of the game as a LP maximization problem.

   (c) Use the LP solver (link will be given) to solve the LP.

4. Suppose two Yoga instructors are offering small personal classes to student groups of size 1, 2 or 3. Each instructor teaches a different style of yoga. The Ashtanga instructor A charges \$10 for one student; but she offers a small discount, charging \$8 per student, if 2 or 3 students enrol. The Bikram instructor B charges \$15 for one student; but he also offers discounted per-student prices of \$12 for two students and \$9 for three students. The prices and discounts are summarized as follows:

Number of Students	1	2	3
A's Per-Student Price	10	8	8
B's Per-Student Price	15	12	9

   Three friends Xavier, Yasher, and Zoe are each considering taking one of the classes. But they each have different preferences or *valuations* for the classes. If they take a class that has valuation v and charges them a price p, then their utility or net *payoff* for that class is $v - p$. Their valuations are as follows:

	Xavier	Yasher	Zoe
Ashtanga	17	16	14
Bikram	20	18	21

   Putting these together, we see that if Xavier takes the Ashtanga class by himself, his net payoff is $17 - 10 = 7$, since his valuation is 17 and he pays the undiscounted price of 10. If one (or both) of his friends took the class with him, his payoff would be 9 (since he would receive the discounted price of 8). Similarly for other students and combinations.

(a) Suppose all three friends must each sign up for one of the classes, simultaneously, without knowledge of what classes their friends are joining. They do know each other's valuations and the prices and discounts available.

(i) Formulate this problem as a matrix game with three players X, Y, Z, with two moves each A and B corresponding to their choice of class. The *payoffs* in the matrix are their net payoffs based on their valuation for the class they join and the (possibly discounted) price they pay. Recall our notation from class for three-player games (see also Exercise 13, Ch.6 of the text): describe the game by specifying *two* matrices, one corresponding to Z (abbreviation for Zoe) selecting A (abbreviation for Ashtanga), the other Z selecting B (for Bikram); each matrix is 2×2, representing the two moves of X (for Xavier) and Y (for Yasher) and each entry in the matrix contains the payoffs for all three players. Write the costs in each cell of the matrix in "player order" X, Y, Z.

Your two matrices should look something like this (with different payoffs x, y, z in each cell of course):

Z: A

	Y:A	Y:B
X:A	x / y / z	x / y / z
X:B	x / y / z	x / y / z

Z: B

	Y:A	Y:B
X:A	x / y / z	x / y / z
X:B	x / y / z	x / y / z

(ii) Which players (if any) have a dominant strategy in this game? Explain.

(iii) What are the pure Nash equilibria of this game? Are any of the pure Nash equilibria Pareto optimal?

(iv) What strategy profile has the highest social welfare (and is it a Nash equilibrium)?

Question 3 continued on next page

(b) Now suppose that the friends each sign up for classes *in sequence*. Once one person signs up for a class, they tweet their choice, so that the remaining friends are informed of this choice before they make their own choices.

(i) Draw the extensive form game tree for this game, assuming that players announce their choices in the following order: first Xavier chooses a class, then Yasher chooses, then Zoe chooses.

(ii) Provide a description of the *subgame perfect equilibrium* in this extensive form game. (This is the equilibrium that is supported by backward induction.) State what actions would be chosen by each player at each stage of the game and what their payoffs will be. Does this correspond to any of the Nash equilibria of the matrix form game?

(iii) Would the outcome of the game change if the order of the players' choices were reversed? Why or why not?

(iv) Suppose that we changed Zoe's valuations as follows: Ashtanga now is worth 15 and Bikram is worth 15. Rewrite the extensive form game with these new payoffs (note: only Zoe's payoffs will change at each leaf of the tree). Describe an equilibrium in this new game that is *not* a subgame perfect equilibrium. Justify your answer. *(Hint: consider how Z might "threaten" X and Y to get a better payoff for herself)*