Homework Assignment 3

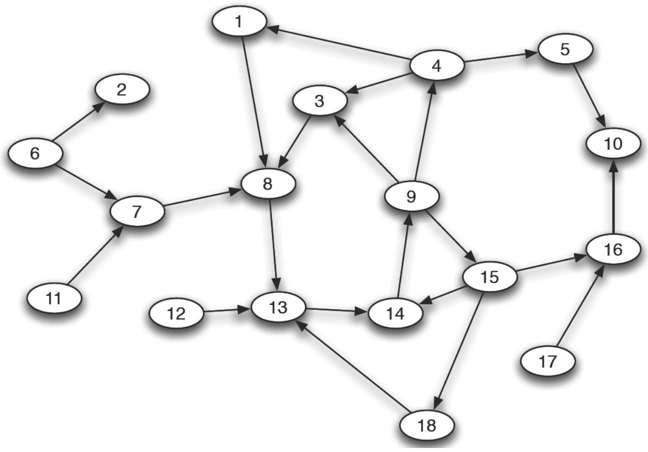
CSC 303: Social and Information Networks

Out: March 11 (first four questions).
Due: March 29, 2019

Be sure to include your name and student number with your assignment. If your handwriting is *possibly* illegible, be sure to hand in your assignment in some typed form. All assignments are to be submitted on Markys by the due date.

You will receive 20% of the points for any (sub)problem for which you write "I do not know how to answer this question." You will receive 10% if you leave a question blank. If instead you submit irrelevant or erroneous answers you will receive 0 points. You may receive partial credit for the work that is clearly "on the right track."

1. Consider the following tiny example of a directed web graph $G = (V, E)$ (this is a reproduction of Fig. 13.8 in the text):



   (a) Suppose we assign all 18 nodes equal initial page rank values of $\frac{1}{18}$. Now suppose we apply the *unscaled* Page Rank algorithm to this graph. After one iteration (or round) of the algorithm, will any nodes have a page rank value of zero? If so, which ones and why? If not, briefly explain why not.

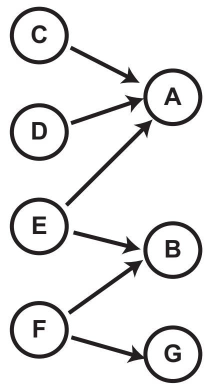
   (b) Answer the same question as in part (a) for both two and three iterations of unscaled page rank.

   (c) When the algorithm converges (i.e., reaches equilibrium), which nodes in the graph will have non-zero page rank values? What will their equilibrium page rank values be? Briefly justify your answer.

   (d) Consider a variation of the graph in part (a) in which a single node can delete a single one of its outgoing edges. Is there any node that can increase its (equilibrium) page rank value by deleting a single outgoing edge? Briefly justify your answer. *(Do not consider simultaneous deletion of multiple edges, just one single edge.)*

   (e) Suppose we use *scaled* Page Rank on the original graph in part (a) with scaling factor $s = 0.9$. Which nodes will have non-zero (equilibrium) page rank values? Qualitatively, briefly describe which node will now have the highest page rank value.

2. The following is web graph with hubs C, D, E and F, and authorities A, B, and G. (This graph and question is a slight modification of Question 3 in Sec. 14.7 of the text.)



   (a) Show the (hubs and authorities) values obtained by running two rounds of the hubs and authorities algorithm on this network. Show the values both before and after the final normalization step, in which we divide each authority score by the sum of all authority scores, and divide each hub score by the sum of all hub scores. You may write the normalized scores as fractions rather than decimals.

   (b) Suppose you wish to add a new web site X to the network and want it to have as high an *authority* score as possible. You also have the ability to create another new web site Y which we can use as a hub to elevate the score of X. Consider the following three options:

      - You add a link from Y to X (and add no other new links).
      - You add a link from Y to A and X (and no other links).
      - You add a link from Y to A, B, G and X (and no other new links).

      For each option, show the normalized authority values that each of A, B, G and X obtain when you run 2-steps of hub-authority computation on the resulting network (as you did in part (a)). Which option gives X the highest (normalized) score?

   (c) (Bonus question) Creating a single hub Y to link to X cannot make X receive the highest authority score, no matter how Y's outlinks are arranged. Instead consider adding three new hubs W, Y, and Z instead of a single new hub. Describe how to arrange the links of these three hubs to authorities so that X receives the highest authority score after two rounds of the algorithm. Hint: all three will need to link to X, but they need not all link to the same authorities.

3. Consider the simple red and blue balls in an urn herding experiment in Chapter 16 of the text and also discussed in the week 7 lectures. Now modify that experiment by assuming that there are four balls in the urn and with probability 1/2 there is three red and one blue ball in the urn (and with probability 1/2, there is three blue and one red ball in the urn). As in the text, we assume each individual hears what the previous individuals report and get their own private signal by randomly drawing one ball from the urn. Suppose the third person hears that the first two invididuals report (blue,blue) and the third individual draws a red ball. What should the third individual report? More specifically, what is the probability that the urn contains three blue and one red ball? You should assume the same tie-breaking rule as in the red and blue balls discussion in the text.

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4. Recall the discussion of self-fulfilling expectations equilibria in a market with positive direct effects in Chapter 17. We considered an individual $x \in [0, 1]$ with a reservation price $r(x)$ for some product; and the direct benefit factor $f(z)$ indicates the relative benefit derived if a fraction z of the population also uses the product. The utility x receives from the product, if fraction z also uses the product, is given by the function $u(x, z) = r(x)f(z)$. Hence x will buy the product if and only if the price is at most $u(x, z)$. Suppose $r(x) = 1 - \frac{3}{4}x$ and $f(z) = z$, so overall utility is $u(x, z) = z(1 - \frac{3}{4}x)$. (This utility function should look familiar from Midterm 2!)

   Now consider a product (like a newspaper) that is purchased every day and where consumers can make decisions each day that are influenced by the number of people who bought the paper the previous day. *Please justify your answer to each of the following questions.* You may find it helpful to sketch a graph of the function $u(z, z)$ on the interval $[0, 1]$.

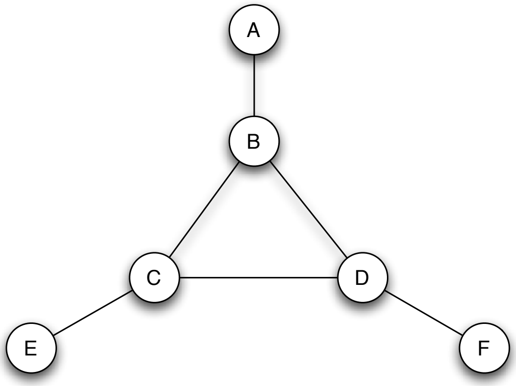
   (a) What is the maximum number of self-fulfilling expectations equilibrium points $z \in [0, 1]$ that can be obtained for any positive price $p^* > 0$.

   (b) Is $z = 1$ a self-fulfilling expectations equilibrium for some price p^*?

   (c) Suppose that some producer (e.g., newspaper publisher) is convinced she can attain an initial market share (say, on the first day of sales) of at least half the consumers. In other words, she believes she can reach an initial market of $z = \frac{1}{2}$. What is the maximum price p^* she can charge so that the *tipping point* does not exceed $z = \frac{1}{2}$?

   (d) Assume the producer adopts the maximum price p^* that sustains the tipping point $z = \frac{1}{2}$, as you computed in part (c). Suppose that initial market z_0 turns out to be $z_0 = \frac{3}{8}$, less than predicted. Given our assumptions regarding who will buy the product the next day based on market share, compute the fraction of people z_1 who will purchase the product the *next* day.

   (e) Suppose the producer gives away the product on certain day $t > 1$, and is able to achieve a market share of $z_t = \frac{5}{8}$. Given our assumptions regarding who will buy the product the next day based on market share, compute the fraction of people z_{t+1} who will purchase the product the *next* day.



5. Consider the 6 node network above where each edge is bidirectional:

   Assume the independent cascade process as in the lectures for weeks 8 and 9 and assume each edge has a one-shot probability $\frac{1}{2}$ of infecting each of its neighbours.

   Suppose A is an initial adopter. What is the probability that node F will become infected at some time.

6. Consider the 5 node network as in Figure 12.2 (d) of the text and the 1-exchange rule experiment as described in the text.

   (a) Describe all the stable outcomes for this network. That is, give a matching and values for the nodes in those matchings. Briefly argue why there cannot be other stable outcomes.

   (b) The text states that experiments show that C will have a little more power than A and E. To what extent do the stable outcomes seem to be inconsistent with this claim? Can you give some explanation for why experimenets can differ in this case from the claim? (Note: there may not be any one best answer here and we will except any plausible answer as to why in experiments C may exhibit slightly more power than A or C.)

7. Consider a stable marriage problem with four women W, X, Y and Z, and four men, E, F, G and H. The women's preference for the men are:

$$X : F \succ H \succ E \succ G \qquad Y : E \succ F \succ G \succ H \qquad Z : E \succ F \succ G \succ H \qquad W : G \succ E \succ F \succ H$$

And the men's preferences for the women are:

$$E : X \succ W \succ Y \succ Z \qquad F : Y \succ Z \succ X \succ W \qquad G : Y \succ Z \succ W \succ X \qquad H : Z \succ W \succ Y \succ X$$

(a) Suppose we run the female-proposing deferred acceptance (FPDA) algorithm. Show how each iteration will proceed by: (i) clearly labelling each iteration; (ii) stating exactly which proposals will be made at that iteration; (iii) and stating exactly which engagements will be in place at the end of that iteration (once relevant proposals are accepted or rejected). Indicate clearly which iteration is the final one and what stable marriage has resulted from FPDA.

(b) We say a man *lies* in FPDA when he rejects a proposal from a woman even though he prefers the proposer to his current fiancee. Identify some man who can *lie* by falsely rejecting a proposal, thereby changing the outcome of FPDA so that he ends up married to a more preferred partner that he did in part (a). State when he should lie in FPDA and what stable marriages will result. (Assume all other men continue to accept and reject proposals truthfully.)