Due: Thursday, February 17 (during reading week)

1. Consider the vertex cover example for which the natural set cover greedy algorithm has an approximation ratio of H_d where d is the maximum degree. In the analysis we allowed the adversary to break ties. Modify that example to show an approximation lower bound for the greedy set cover algorithm when allowing the algorithm to break ties (if any exist).

2. We consider the general knapsack problem where we represent the input as $\{\mathcal{I}; B\} = \{(b_1, v_1), (b_2, v_2), \ldots, (b_n, v_n); B\}$. Here b_i is the "bulk" of item i , v_i is the value of item i and B is the knapsack bound (on the total bulk). Without loss of generality we assume that $b_i \leq B$ for all i.

   Consider the following algorithm $Brute-greedy_k$ for the general knapsack problem:

   For every $S \subseteq \mathcal{I}$ such that $|S| \leq k$ and $\sum_{i \in S} b_i \leq B$, we compute the feasible solution $Greedy_S$ as follows: Place S in the knapsack and then greedily (using the ordering determined by non increasing v_i/b_i) add items in $\mathcal{I} - S$ to the knapsack. $Brute - greedy_k$ returns the $Greedy_S$ which has the largest profit. Show that $Brute - greedy_k$ is a $\frac{k}{k+1}$ approximation algorithm.

3. Some research questions to think about:

   (a) Can the JBAP problem (or any of the subcases discussed in class) be solved by a fixed order local ratio (stack) algorithm using an ordering different from $f_1 \leq f_2 \leq \ldots f_n$? For example what about $v_1/p_1 \leq v_2/p_2 \leq \ldots v_n/p_n$ where $p_i = f_i - s_i$ or $v_1/b_1 \leq \ldots \leq v_n/b_n$.

   (b) In class we showed that the local ratio algorithm of Bar Noy et al cannot solve the BAP problem with approximation ratio $\leq \frac{1+\frac{1}{k-1}}{2} \leq 1/2$ for the case that $b_i \leq 1/k$ for all i. Can this result be extended to any fixed order stack algorithm which orders the input intervals so that $f_1 \leq f_2 \leq \ldots f_n$?

   (c) For the general knapsack problem, what is the best negative result you can prove for the approximation ratio of any i) fixed order stack algorithm and/or ii) any one pass algorithm with revocable acceptances?

   (d) Can you prove any negative result for the approximation ratio of adaptive stack algorithms for the JBAP problem (or any of the subcases discussed in class)?