CSC 2421S 2003, Assignment 2 Due: Feb March 25

1. In the analysis of the primal dual algorithm for the Steiner Forest problem, we used the fact that if C is a component with respect to the currently chosen edges and $f(C) = 0$, then $deg_{F'}(C) \neq 1$. Here $f(C) = 0$ iff there does not exist $u_i, v_i \in S_i$ and $u_i \in C, v_i \notin C$ and F' is the set of edges (i.e. the Steiner Forest) produced by the algorithm.

   Constuct an input instance where some such C results during the execution of the algorithm with $deg_{F'}(C) > 1$.

2. Consider the scheduling problem where each job J_j is characterized by a deadline d_j, a processing time p_j and a profit/weight w_j. The objective is to schedule a subset of jobs on one machine (without any overlap) so as to maximize the profit of the jobs scheduled. Show how to use dynamic programming and scaling to obtain a FPTAS for this problem. Hint: consider algorithm 8.2 in Vazirani and note that a set of jobs can be scheduled iff they can be scheduled in order of their deadlines.

3. Consider the classical makespan scheduling problem $P| |C_{max}$ where all jobs must be scheduled on a set of identical machines so as to minimize the maximum load on any machine (= the makespan) . Consider the following local search algorithm: A *move* operation moves a job from one machine to another. A machine is called critical if its load is equal to the current makespan. A greedy move is one that either reduces the current makespan or maintains the current makespan but reduces the number of critical machines. The local search algorithm continues to apply greedy moves until no such move exists. Show that the approximation ratio of this local search algorithm is exactly $2 - \frac{2}{m+1}$ where m is the number of machines. (Note that for all m, this local search algorithm beats the performance of Graham's greedy list scheduling algorithm.)